Bose-Einstein Condensates and Detection Operators

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Abstract

These are rough, unfinished and unfocused notes about Bose-Einsten condensates and detection operators. They contain questions I have posed to myself and so far not got round to doing anything about answering them. Comments and remarks are most welcome — please contact me *via* my web site.

1. INTRODUCTION

Two Bose-Einstein condensates that "leak" atoms to a dectector are treated as two harmonic oscillators. In Section 2 the essential harmonic oscillator problem is introduced in terms of two sets of oscillators $(\pm)$. A physical meaning can be attached to the $(+)$ oscillator: can one be given to the other and what do both say about BEC?
A short discussion about compling an oscillator to a set of continuum modes is in Section 5

2. DETECTION OPERATORS

Introduce operators

$$\hat{A}_{\pm}(\phi) = 2^{-1/2}\left\{\hat{a} \pm \hat{b}\exp(i\phi)\right\}, \tag{1a}$$
$$\hat{A}^{\dagger}_{\pm}(\phi) = 2^{-1/2}\left\{\hat{a}^{\dagger} \pm \hat{b}^{\dagger}\exp(-i\phi)\right\}, \tag{1b}$$

where ϕ is some parameter, and

$$\hat{a} = 2^{-1/2}\left\{\hat{A}_{-}(\phi) + \hat{A}_{+}(\phi)\right\}, \tag{2a}$$
$$\hat{b} = 2^{-1/2}\exp(-i\phi)\left\{\hat{A}_{-}(\phi) - \hat{A}_{+}(\phi)\right\} \tag{2b}$$

are the annihilation components of two independent sets of Boson operators

$$\left[\hat{a}, \hat{a}^{\dagger}\right] = \left[\hat{b}, \hat{b}^{\dagger}\right] = 1, \tag{3a}$$
$$\left[\hat{a}, \hat{b}^{\dagger}\right] = 0, \tag{3b}$$

associated with the eigen-number equations

$$\hat{a}^{\dagger}\hat{a}|\alpha\rangle = \alpha|\alpha\rangle, \tag{4a}$$
$$\hat{b}^{\dagger}\hat{b}|\beta\rangle = \beta|\beta\rangle. \tag{4b}$$

If the oscillators are condensates then α, β are the numbers of atoms in each. The operators (1) evaluated at the same value of ϕ are also Bosonic, as indicated by the commutator

$$\left[\hat{A}_{\alpha}(\phi), \hat{A}^{\dagger}_{\alpha'}(\phi)\right] = \delta_{\alpha\alpha'}, \qquad \alpha, \alpha' = \pm. \tag{5}$$

The operator $\hat{A}_{+}$ has been used as a dectector annihilation operator for two Bose-Einstein condensates [1], where ϕ is associated with the detector's position. Can a physical meaning can be attached to $\hat{A}_{-}$?
If the states of the oscillators $(\pm)$ are $|\psi_{\pm}\rangle$ then the probabilities

$$P_{\pm}(\phi) = \langle\psi_{\pm}|\hat{A}^{\dagger}_{\pm}(\phi)\hat{A}_{\pm}(\phi)|\psi_{\pm}\rangle \tag{6}$$

of detecting a mode may be written as

$$P_{\pm}(\phi) = \frac{1}{2}\langle\psi_{\pm}|\left\{\hat{n} \pm \hat{m}(\phi)\right\}|\psi_{\pm}\rangle, \tag{7}$$

where the Hermitian operators

$$\begin{aligned}
\hat{n} &= \hat{a}^{\dagger}\hat{a} + \hat{b}^{\dagger}\hat{b} \\
&= \hat{A}^{\dagger}_{+}(\phi)\hat{A}_{+}(\phi) + \hat{A}^{\dagger}_{-}(\phi)\hat{A}_{-}(\phi), && \text{(8a)} \\
\hat{m}(\phi) &= \hat{a}^{\dagger}\hat{b}\exp(i\phi) + \hat{a}\hat{b}^{\dagger}\exp(-i\phi) \\
&= \hat{A}^{\dagger}_{+}(\phi)\hat{A}_{+}(\phi) - \hat{A}^{\dagger}_{-}(\phi)\hat{A}_{-}(\phi) && \text{(8b)}
\end{aligned}$$

are linear combinations of (2). The states $|\psi_{-}\rangle$ of the oscillator $(-)$ are separate from $|\psi_{+}\rangle$, the states of the oscillator $(+)$. Perhaps the detection probability should involve product states $|\psi\rangle = |\psi_{-}\rangle|\psi_{+}\rangle$.
Consider the expectation values

$$\langle\psi_{\pm}|\hat{m}(\phi)|\psi_{\pm}\rangle = \langle\psi_{\pm}|\left\{\hat{a}^{\dagger}\hat{b}\exp(i\phi) + \hat{b}^{\dagger}\hat{a}\exp(-i\phi)\right\}|\psi_{\pm}\rangle$$
$$= \left\{\langle\psi_{\pm}|\hat{a}^{\dagger}\hat{b}|\psi_{\pm}\rangle + \langle\psi_{\pm}|\hat{a}^{\dagger}\hat{b}|\psi_{\pm}\rangle^{*}\right\}\cos\phi + i\left\{\langle\psi_{\pm}|\hat{a}^{\dagger}\hat{b}|\psi_{\pm}\rangle - \langle\psi_{\pm}|\hat{a}^{\dagger}\hat{b}|\psi_{\pm}\rangle^{*}\right\}\sin\phi \quad . \tag{9}$$

If $\langle\psi_{\pm}|\hat{a}^{\dagger}\hat{b}|\psi_{\pm}\rangle$ is a real number (positive or negative) then Eq. (9) simplifies to

$$\langle\psi_{\pm}|\hat{m}(\phi)|\psi_{\pm}\rangle = 2\cos\phi\left|\langle\psi_{\pm}|\hat{a}^{\dagger}\hat{b}|\psi_{\pm}\rangle\right|, \tag{10}$$

where the use of the modulus seems to take care of the possibility that the signs of $\langle\psi_{\pm}|\hat{a}^{\dagger}\hat{b}|\psi_{\pm}\rangle$ may be altered by the transformations

$$\hat{a} \rightarrow \pm i\hat{a}, \qquad \hat{b} \rightarrow \mp i\hat{b}. \tag{11}$$

However, the use of the modulus may be too restrictive since the transformation (11) results in $\hat{A}_{+}(\phi) \rightarrow \pm i\hat{A}_{-}(\phi)$, $\hat{A}_{-}(\phi) \rightarrow \pm i\hat{A}_{+}(\phi)$. This is equivalent to $\hat{A}^{\dagger}_{+}(\phi)\hat{A}_{+}(\phi) \rightarrow \hat{A}^{\dagger}_{-}(\phi)\hat{A}_{-}(\phi)$. The modulus sign will be kept for the time being, but this will need to be reviewed; especially since its use suggests below the existence of both maximum and minimum probabilities. With this proviso in mind, the probabilities (7) may be written in general as

$$\begin{aligned}
P_{\pm}(\phi) &= \frac{\langle\psi_{\pm}|\hat{n}|\psi_{\pm}\rangle}{2} \\
&\times \left\{1 \pm 2\cos(\phi-\theta)\frac{\left|\langle\psi_{\pm}|\hat{a}^{\dagger}\hat{b}|\psi_{\pm}\rangle\right|}{\langle\psi_{\pm}|\hat{n}|\psi_{\pm}\rangle}\right\},
\end{aligned} \tag{12}$$

where the parameter θ has been introduced to allow explicit determinations of the turing points of $P_{\pm}(\phi)$. Differentiating (12) with respect to ϕ once, and then twice, leads to the conclusion that $P_+(\theta)$ corresponds to a maximum probability and $P_-(\theta)$ corresponds to a minimum probability. This suggests that $P_+(\phi)$ is to be interpretated as a probability distribution containing a maximum probability, while the distribution $P_-(\phi)$ is one that contains a minimum probability. To calculate the specific values of these probabilities, Eq. (6) needs to be normalised.

It might seem that the probabilities may be normalised according to the scheme

$$\int_{\phi_1}^{\phi_2} d\phi \; P_\pm(\phi) = 1, \tag{13}$$

where ϕ_1, ϕ_2 are the end-points of interest. That is, the prescription is to be regarded as two separate equations: one for $P_-(\phi)$ and one for $P_+(\phi)$. This leads to the equations

$$1 = \frac{\langle\psi_\pm|\hat{n}|\psi_\pm\rangle}{2}\,(\phi_1 - \phi_2) \pm \left|\langle\psi_\pm|\hat{a}^\dagger\hat{b}|\psi_\pm\rangle\right| \{\sin(\phi_2 - \theta) - \sin(\phi_1 - \theta)\}\,. \tag{14}$$

The simplest choices for the end points are perhaps $\phi_2 = \pi + \theta$ and $\phi_1 = -\pi + \theta$. These give

$$\langle\psi_\pm|\hat{n}|\psi_\pm\rangle = \pi^{-1}, \tag{15}$$

and, therefore,

$$P_\pm(\phi) = \frac{1}{2\pi}\left\{1 \pm 2\pi\cos(\phi - \theta)\left|\langle\psi_\pm|\hat{a}^\dagger\hat{b}|\psi_\pm\rangle\right|\right\} \tag{16}$$

is the probability of detecting a mode. Eq. (16) leads to

$$P_+(\theta) = \frac{1}{2\pi}\left\{1 + 2\pi\left|\langle\psi_+|\hat{a}^\dagger\hat{b}|\psi_+\rangle\right|\right\}, \tag{17a}$$

$$P_-(\theta) = \frac{1}{2\pi}\left\{1 - 2\pi\left|\langle\psi_-|\hat{a}^\dagger\hat{b}|\psi_-\rangle\right|\right\} \tag{17b}$$

as the maximum and minimum values of the probabilities. If $P_+(\theta)$ were to be unity then $\left|\langle\psi_+|\hat{a}^\dagger\hat{b}|\psi_+\rangle\right|$ would equal $1 - (1/2\pi)$. Similarly, a vanishing $P_-(\theta)$ would correspond to the expectation value $\left|\langle\psi_-|\hat{a}^\dagger\hat{b}|\psi_-\rangle\right|$ equal to $1/2\pi$. Different values of ϕ_1, ϕ_2 may be chosen.

3. BASIS STATES

The operators (8) possess a common basis since

$$[\hat{n}, \hat{m}(\phi)] = 0 \tag{18}$$

for all values of ϕ. Perhaps this basis ought to be a function of the parameter ϕ: it will be therefore be written as $|f_{n,m}(\phi)\rangle$; where n, m are the eigenvalues of the operators (8), according to the eigenvalue equations

$$\hat{n}|f_{n.m}(\phi)\rangle = n|f_{n,m}(\phi)\rangle, \tag{19a}$$

$$\hat{m}|f_{n.m}(\phi)\rangle = m|f_{n,m}(\phi)\rangle. \tag{19b}$$

The quantum number

$$n = \alpha + \beta \tag{20}$$

is related to the eigenvalues defined in Eq. (4). How do the operators (1) act on this basis? This can be answered from the commutators

$$\left[\hat{n}, \hat{A}_{\pm}(\phi)\right] = -\hat{A}_{\pm}(\phi), \tag{21a}$$

$$\left[\hat{m}, \hat{A}_{\pm}(\phi)\right] = \mp\hat{A}_{\pm}(\phi). \tag{21b}$$

Therefore, both the annihilation operators $\hat{A}_{\pm}(\phi)$ lower the quantum number n by unity, but $\hat{A}_{+}(\phi)[\hat{A}_{-}(\phi)]$ decreases [increases] m by unity. This, and Eqs. (8), (19), are consistent with the prescription

$$\hat{A}_{\pm}(\phi)|f_{n,m}(\phi)\rangle = 2^{-\frac{1}{2}}(n \pm m)^{\frac{1}{2}}|f_{n-1,m\mp1}(\phi)\rangle, \tag{22a}$$

$$\hat{A}^{\dagger}_{\pm}(\phi)|f_{n,m}(\phi)\rangle = 2^{-\frac{1}{2}}(n \pm m + 2)^{\frac{1}{2}}|f_{n+1,m\pm1}(\phi)\rangle. \tag{22b}$$

It is evident from (22) that the quantum numbers n, m associated with the $(-)$ oscillator $\hat{A}_{-}(\phi)$, $\hat{A}^{\dagger}_{-}(\phi)$ are restricted in the sense that m cannot be greater than n. What does this mean if n, m are taken as the numbers of atoms in two condensates? Equation (22a) implies that $|f_{n,n}(\phi)\rangle$ is a ground state of the $(-)$ oscillator for all n since it is annihilated by the action of $\hat{A}_{-}(\phi)$. Something very similar occurs elsewhere [2].
The eigenstates can be generated from $|f_{0,0}(\phi)\rangle$ by repeated applications of the $\pm$ creation operators, according to

$$\left[\hat{A}^{\dagger}_{-}(\phi)\right]^{p}|f_{0,0}(\phi)\rangle = \sqrt{p!}|f_{p,-p}(\phi)\rangle, \tag{23a}$$

$$\left[\hat{A}^{\dagger}_{+}(\phi)\right]^{q}|f_{0,0}(\phi)\rangle = \sqrt{q!}|f_{q,q}(\phi)\rangle, \tag{23b}$$

where p, q are integers. These equations can be combined to give

$$[q!p!]^{-\frac{1}{2}}\left[\hat{A}^{\dagger}_{-}(\phi)\right]^{q}\left[\hat{A}^{\dagger}_{+}(\phi)\right]^{p}|f_{0,0}(\phi)\rangle = |f_{n,m}(\phi)\rangle, \tag{24a}$$

$$q = (n-m)/2 = n_{-}, \tag{24b}$$

$$p = (n+m)/2 = n_{+}, \tag{24c}$$

providing the restriction (24b) is in place. The parameters $n_{\pm}$ are the eigenvalues of the equations

$$\hat{A}^{\dagger}_{\pm}(\phi)\hat{A}_{\pm}(\phi)|\,\rangle = n_{\pm}|\,\rangle, \tag{25}$$

where the eigenfunctions are conveniently written as $|\,\rangle$.
The ground state

$$|f_{0,0}(\phi)\rangle = |0\rangle_{\alpha}|0\rangle_{\beta}g(\phi), \tag{26}$$

where $|0\rangle_\alpha$ [$|0\rangle_\beta$] is the ground state of the oscillator $\hat{a}$, $\hat{a}^\dagger$ [$\hat{b}$, $\hat{b}^\dagger$]. The function $g(\phi)$ would seem to be necessary in order to introduce some ϕ dependence. If now Eqs. (1b), (26) are subsituted into (24a) then

$$|f_{n,m}(\phi)\rangle = [2^n q!p!]^{-\frac{1}{2}} \left[\hat{a}^\dagger\right]^n \{1 - \hat{x}\exp(-i\phi)\}^q \times \{1 + \hat{x}\exp(-i\phi)\}^p g(\phi)|0\rangle_\alpha|0\rangle_\beta, \tag{27}$$

where

$$\hat{x} = \hat{b}^\dagger \left[\hat{a}^\dagger\right]^{-1} \tag{28}$$

and the fact that $n = p + q$ have been used. Assume that polynomial relationship

$$(1-x)^q(1+x)^p = \sum_{k=0}^{q+p} U_k x^k \tag{29}$$

can be applied to operators. Therefore, Eq. (27) becomes

$$\begin{aligned} |f_{n,m}(\phi)\rangle &= [2^n q!p!]^{-\frac{1}{2}} g(\phi) \sum_{k=0}^{n} U_k \left[\hat{a}^\dagger\right]^{n-k} \left[\hat{b}^\dagger\right]^k \exp(-ik\phi)|0\rangle_\alpha|0\rangle_\beta \\ &= [2^n q!p!]^{-\frac{1}{2}} g(\phi) \sum_{k=0}^{n} U_k [k!(n-k)!]^{\frac{1}{2}} \exp(-ik\phi)|n-k\rangle_\alpha|k\rangle_\beta. \end{aligned} \tag{30}$$

Using the expansions

$$(1 \pm x)^r = 1 \pm \binom{r}{1} x + \binom{r}{2} x^2 \pm \binom{r}{3} x^3 + \ \ldots, \tag{31}$$

where

$$\binom{r}{n} = \frac{r(r-1)(r-2)\ldots(r-n+1)}{n!}, \tag{32}$$

the coefficient

$$\begin{aligned} U_k &= \binom{q}{0}\binom{p}{k} - \binom{q}{1}\binom{p}{k-1} + \binom{q}{2}\binom{p}{k-2} - \binom{q}{3}\binom{p}{k-3} + \ \ldots \\ &+ (-1)^{k-2}\binom{q}{k-2}\binom{p}{2} + (-1)^{k-1}\binom{q}{k-1}\binom{p}{1} + (-1)^k\binom{q}{k}\binom{p}{0} \end{aligned} \tag{33}$$

may be determined. By convention,

$$\binom{q}{0} = \binom{p}{0} = 1. \tag{34}$$

Consequently, U_k is real.
If $q = 0$, in other words $m = n$, then Eq. (33) simplifies to

$$U_k = \binom{p}{k}, \qquad q = 0, \tag{35}$$

giving

$$|f_{n,n}(\phi)\rangle = \left[\frac{n!}{2^n}\right]^{\frac{1}{2}} g(\phi)\sum_{k=0}^{n}[k!(n-k)!]^{-\frac{1}{2}} \times \exp(-ik\phi)|n-k\rangle_\alpha|k\rangle_\beta. \quad (36)$$

4. EXPECTATION VALUES

The equation

$$\hat{a}^\dagger\hat{b}|f_{n,m}(\phi)\rangle = [2^n q!p!]^{-\frac{1}{2}}\, g(\phi)\sum_{k=0}^{n} U_k[k!k(n-k)!(n-k+1)]^{\frac{1}{2}}\exp(-ik\phi)|n-k+1\rangle_\alpha|k-1\rangle_\beta \quad (37)$$

allows a determination of the expectation value

$$\langle f_{n,m}(\phi)|\hat{a}^\dagger\hat{b}|f_{n,m}(\phi)\rangle = [2^n q!p!]^{-1}\,|g(\phi)|^2\sum_{k=0}^{n} U_k U_{k+1}[k!k(n-k+1)!]^{\frac{1}{2}}[(k+1)!(k+1)(n-k)!]^{\frac{1}{2}} \quad (38)$$

to be made. The Gaussian approximation

$$\binom{x}{m} \sim 2^x\sqrt{\frac{2}{\pi x}}\exp\left[-\frac{2}{x}\left(m-\frac{x}{2}\right)^2\right] \quad (39)$$

for a binomial coefficient may be used.

5. COMPLEX INTERACTION STRENGTH

The most obvious difference between input-output theory for the atomic and optical cases is that the energy spectrum of the free atom is quadratic in momentum, rather than linear. Another important difference is to allow for a complex interaction between the single mode of the cavity and the outside world. A complex interaction allows for a "larger class of interaction" [3] to be considered.
An oscillator (the condensate) is coupled to a continuum. Write the Hamiltonian as

$$\hat{H} = k_o^2\hat{a}^\dagger\hat{a} + \int_{-\infty}^{\infty} dk\; k^2\hat{b}(k)^\dagger\hat{b}(k) + \hat{H}_{\rm int}, \quad (40)$$

where the interaction

$$\hat{H}_{\rm int} = \int_{-\infty}^{\infty} dk\;\left\{V(k)\hat{b}(k)\hat{a}^\dagger + V^*(k)\hat{a}\hat{b}^\dagger(k)\right\} \quad (41)$$

is written in terms of an interaction strength

$$v(k) = |v(k)|\exp[i\theta(k)] \quad (42)$$

that is taken to be complex. Introduce new operators

$$\hat{c}(k) = N(k)\left\{|v(k)|\hat{b}(k) + |v(-k)|\hat{b}(-k)\right\}, \quad (43a)$$

$$\hat{d}(k) = M(k)\left\{-|v(-k)|\hat{b}(k) + |v(k)|\hat{b}(-k)\right\} \quad (43b)$$

as functions of positive k. Note that these operators are slightly different from those in [4] in that they involve the modulus of the interaction. The parameters $M(k)$, $N(k)$ are such that their moduli must be equal in order that the commutators

$$\left[\hat{c}(k), \hat{c}^\dagger(k')\right] = \left[\hat{d}(k), \hat{d}^\dagger(k')\right] = \delta(k - k') \quad (44)$$

between the operators (43) and their Hermitean conjugates are canonical. This follows from the fact that the requirement (44) implies the relationship

$$|N(k)|^2 = |v(k)|^2 + |v(-k)|^2 = |M(k)|^2. \quad (45)$$

The commutator between $\hat{c}(k)$ and $\hat{d}^\dagger(k')$ vanishes indentically.
If Eqs. (43) are re-arranged to give

$$\hat{b}(-k) = \frac{1}{|v(k)|^2 + |v(-k)|^2}\left\{\frac{|v(-k)|}{N(k)}\hat{c}(k) + \frac{|v(k)|}{M(k)}\hat{d}(k)\right\}, \quad (46a)$$

$$\hat{b}(k) = \frac{1}{|v(k)|^2 + |v(-k)|^2}\left\{\frac{|v(k)|}{N(k)}\hat{c}(k) - \frac{|v(-k)|}{M(k)}\hat{d}(k)\right\} \quad (46b)$$

then it is apparent that differentiating between N, M simply results in a phase shift to the operator $\hat{d}(k)$. Therefore,

$$\hat{b}(-k) = \frac{|N(k)|^2}{N(k)}\left\{|v(-k)|\hat{c}(k) + |v(k)|\hat{d}(k)\right\}, \quad (47a)$$

$$\hat{b}(k) = \frac{|N(k)|^2}{N(k)}\left\{|v(k)|\hat{c}(k) - |v(-k)|\hat{d}(k)\right\}, \quad (47b)$$

$$|N(k)|^{-2} = |v(k)|^2 + |v(-k)|^2 \quad (47c)$$

may be chosen as the most general forms of the operators. Equations (47) allow the continuum oscillator

$$\int_{-\infty}^{\infty} dk\; k^2 \hat{b}^\dagger(k)\hat{b}(k) = \int_0^\infty dk\; k^2 \left\{\hat{c}^\dagger(k)\hat{c}(k) + \hat{d}^\dagger(k)\hat{d}(k)\right\} \quad (48)$$

to be written as before, in terms of two oscillators defined only in positive k-space. However, the operator $\hat{d}$ ($\hat{d}^\dagger$) does not couple to $\hat{a}^\dagger$ ($\hat{a}$) and the interaction (41) may be written as

$$\hat{H}_{\text{int}} = \int_0^\infty dk\; \lambda(k)\left\{\hat{c}(k)\hat{a}^\dagger + \hat{a}\hat{c}^\dagger(k)\right\}, \quad (49a)$$

$$\lambda(k) = 1/|N(k)| \quad (49b)$$

only if the phases of (42) are identical for positive and negative k. That is, only if

$$\theta(k) = \theta(-k). \tag{50}$$

It might seem therefore that the presence of a single level, separate from the continuum, a claim made by [4], is justified only in the special case of (50). But then how special is (50)?

6. ROTATING WAVE APPROXIMATION

The rotating wave approximation is implicit in the form of the interaction (41). If the output coupling mechanism is through a Raman transistion — a $\mathbf{d} \cdot \mathbf{E}$ interaction — then counter-rotating terms would be present. In optics, the use of the approximation is justified only in the region of resonance [5]. In coupling a single oscillator of "frequency" k_o^2 to a continuum of oscillators with an infinite range of "frequencies" k^2 can this approximation be justified?
If counter-rotating terms $\hat{b}(k)\hat{a}$, $\hat{a}^\dagger\hat{b}^\dagger(k)$ were to be included in the interaction then providing Eq. (50) is again satisfied then $\hat{d}$ ($\hat{d}^\dagger$) does not couple to $\hat{a}$, ($\hat{a}^\dagger$).
It is however possible to formally eliminate the counter-rotating terms. Thus, modifying slightly the reults from [5], one can derive

$$\hat{H}_{\text{int}} = \int_{-\infty}^{\infty} dk \, \frac{2k_o^2}{k_o^2 + k^2} \left\{ V(k)\hat{b}(k)\hat{a}^\dagger + V^*(k)\hat{a}\hat{b}^\dagger(k) \right\}, \tag{51}$$

as the interaction without using the rotating wave approximation. This leads to

$$\hat{H}_{\text{int}} = \int_{0}^{\infty} dk \, \frac{2\lambda(k)k_o^2}{k_o^2 + k^2} \left\{ \hat{c}(k)\hat{a}^\dagger + \hat{a}\hat{c}^\dagger(k) \right\}, \tag{52}$$

where $\lambda(k)$ is given by (49b) Again, Eq. (50) must be satisfied. Since terms in $\hat{a}^2$ and $\hat{b}^2(k)$ are ignored, the operators in (52) are not transformed.
It is said that the use of the rotating wave approximation is justified on the grounds that no atoms are destroyed. WHY?

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