Reports, Research, Consulting

Colin-Baxter.com

Market-Relevant Science

Radiation Pressure and Momentum Transfer in Dielectrics: The Photon Drag Effect

Rodney Loudon, Stephen M. Barnett, C. Baxter

Physical Review A, **71**, 063802 (2005)

Front Cover

PHYSICAL REVIEW A **71**, 063802 (2005)

Radiation pressure and momentum transfer in dielectrics: The photon drag effect

Rodney Loudon
Electronic Systems Engineering, University of Essex, Colchester C04 3SQ, United Kingdom

Stephen M. Barnett and C. Baxter
Department of Physics, University of Strathclyde, Glasgow G4 0NG, United Kingdom
(Received 28 December 2004; published 10 June 2005)

The momentum transfer from light to a dielectric material in the photon drag effect is calculated by evaluation of the relevant Lorentz force. In accordance with measurements on Si and Ge, the material is taken as a two-component optical system, with charge carriers described by an extinction coefficient κ in a host semiconductor described by real refractive indices η_p (phase) and η_g (group). The calculated momentum transfer to the charge carriers alone has the value $\eta_p\hbar\omega/c$ per photon, the so-called Minkowski value, found experimentally. The time-dependent Lorentz force is calculated for light in the form of a narrow-band single-photon pulse. When the pulse is much shorter than the attenuation length, which is much shorter than the sample thickness, there is a clear separation in time between surface and bulk contributions to the forces. The total bulk momentum transfer (charges plus host) in this case is found to be $\hbar\omega/\eta_g c$, the so-called Abraham value.

DOI: 10.1103/PhysRevA.71.063802 PACS number(s): 42.50.Nn

I. INTRODUCTION

Theories of radiation pressure and the momentum of light in dielectrics have frequently involved arguments about the correct form of the stress tensor of the electromagnetic field in a material medium (see [1] for a review). Several formulations of the stress tensor have been developed, but particular attention has been given to the two contenders in the so-called Abraham-Minkowski controversy. The quantity at issue is the momentum per photon available for transfer from the light to the medium. Most theoretical work is concerned with nondispersive media, but the more general calculations presented here allow for material dispersion, with the phase refractive index η_p different from the group refractive index η_g. The accepted convention associates the term *Abraham momentum* with the quantity $\hbar\omega/\eta_p c$ or $\hbar\omega/\eta_g c$ and the term *Minkowski momentum* with the quantity $\eta_p\hbar\omega/c$. It is convenient to follow this nomenclature, with *Abraham* and *Minkowski* used as labels for these forms of momentum transfer. The few measurements of radiation pressure in dielectric media appear to support the Minkowski value for the momentum transfer, and several calculations strive to produce this result from theories that obstinately seem to support the Abraham expression.

Observations of radiation pressure in media rely on measurements of the forces exerted by light on some distinguishable component of the medium itself or on some object immersed in the medium. We take the view that the Lorentz force provides the fundamental description of radiation pressure effects and use it as the basis for the present calculations. This same approach has been used previously [2,3] in treatments of the radiation pressure on a general semi-infinite dielectric. It has the advantage that no prior assumptions are made about the magnitude of the optical momentum in the medium. However, the results of the calculation do provide information on the transfer of momentum per photon to the observed object. Moreover, the calculated force represents the measured quantity, and not some subsidiary quantity that may not itself be directly measurable. As for the "controversy" between different formulations of electromagnetic theory, we believe that all formulations are equally valid and that they produce the same predictions when properly applied to specific problems. Any controversy arises from the improper isolation of momentum densities from other contributing terms in the relevant continuity equations.

The photon drag effect, observed in 1970 [4,5], is one of the simplest manifestations of radiation pressure. The main effect is the generation of currents or electric fields in semiconductors, notably germanium and silicon, by the transfer of momentum from an incident light beam to the charge carriers. The requirements of energy and momentum conservation generally forbid the absorption of photons by free carriers, and the process can only take place by interband transitions or with the assistance of phonon absorption or emission. The magnitude and sign of the transfer depend on the detailed band structure of the material in complicated ways [6]. However, for optical angular frequencies ω and charge carrier relaxation times τ sufficiently small that $\omega\tau \ll 1$, the details of the band structure become irrelevant [7]. The momentum transfer to the charge carriers was observed to approach a value given by $\eta_p\hbar\omega/c$ per photon for sufficiently long optical wavelengths, although the conditions of the experiments were such that the phase and group refractive indices could not be distinguished. This same form of limiting momentum transfer was found experimentally for both *n*- and *p*-type germanium and silicon [7]. The observed effect of the material is thus to multiply the free-space photon momentum $\hbar\omega/c$ by an additional factor η_p, and this was taken as evidence in support of the Minkowski momentum transfer.

The calculations presented here show that the transfer of momentum to the charge carriers is accompanied by an additional transfer of momentum to the host semiconductor. The outstanding advantage of the photon drag measurements is their ability to separate the contributions from the momentum transfer to the charge carriers, associated essentially with the extinction coefficient κ, from the unobserved momentum transfer to the host material, associated with the real refractive index η_p. The calculated momentum transfer to the charge carriers has the Minkowski value of $\eta_p\hbar\omega/c$ per photon, in agreement with experiment. Calculations equivalent to those reported here could also be performed in the framework of classical electromagnetic theory, with similar conclusions. However, the quantum theory is easily applied and it has the advantage of providing results directly expressed in terms of momentum transfers per incident photon.

The general formulations of the electromagnetic energy-momentum stress tensor by Abraham [8], Minkowski [9], and others are discussed in Sec. II, where the equivalence of the superficially different theories for the photon drag effect is demonstrated. The relevant optical properties of semiconductors are summarized in Sec. III, and the Lorentz forces on the charge carriers and host semiconductor are calculated in Sec. IV. The time dependences of the forces on the two components produced by a narrow-band single-photon pulse are calculated in Sec. V, and the results are approximated for some limiting values of pulse length, attenuation length, and sample thickness. In particular, when the lengths as listed are in ascending orders of magnitude, it is shown that the bulk material acquires an Abraham total momentum transfer of $\hbar\omega/\eta_g c$ per photon. Our conclusions are summarized in Sec. VI.

II. ELECTROMAGNETIC STRESS TENSORS

A. History

Two varieties of stress tensor for the electromagnetic field in a material medium were formulated early in the last century by Abraham [8] and Minkowski [9]. This section is devoted to a brief review of these formulations and others, in order to provide a background for the results on the photon drag effect derived in the following sections. These results are based on evaluations of Lorentz forces and are independent of any specific formulation. However, it is emphasized here that the Abraham and Minkowski theories are equally valid when they are properly applied to specific problems. The refractive index η and the electric susceptibility $\chi=\eta^2-1$ are taken to be real and independent of frequency for the calculations of this section.

The stress tensors appear in equations expressing linear momentum conservation, which are often written as

$$\frac{\partial g_A^j}{\partial t}+\sum_{i=1}^{3}\frac{\partial T_A^{ij}}{\partial x^i}=-f_A^j \text{ and } \frac{\partial g_M^j}{\partial t}+\sum_{i=1}^{3}\frac{\partial T_M^{ij}}{\partial x^i}=0, \quad (2.1)$$

where the subscripts A and M indicate Abraham and Minkowski. The tensorial components of the momentum flux density of the light beam are [1]

$$T_A^{ij}=\frac{1}{2}\{\delta^{ij}\mathbf{E}\cdot\mathbf{D}-E^iD^j-E^jD^i\}+\frac{1}{2}\{\delta^{ij}\mathbf{H}\cdot\mathbf{B}-H^iB^j-H^jB^i\}$$

$$T_M^{ij}=\frac{1}{2}\delta^{ij}\mathbf{E}\cdot\mathbf{D}-E^iD^j+\frac{1}{2}\delta^{ij}\mathbf{H}\cdot\mathbf{B}-H^iB^j, \quad (2.2)$$

where the A form is symmetric in the spatial coordinates i, $j=1, 2, 3$ but the M form is not. The dependence of the field vectors on space and time is not shown explicitly. The momentum densities are

$$\mathbf{g}_A=\mathbf{E}\times\mathbf{H}/c^2 \text{ and } \mathbf{g}_M=\mathbf{D}\times\mathbf{B} \quad (2.3)$$

and the so-called Abraham force density is

$$\mathbf{f}_A=\varepsilon_0\chi\frac{\partial}{\partial t}(\mathbf{E}\times\mathbf{B}). \quad (2.4)$$

The association of each tensor with a different momentum density has been regarded historically as significant [1], as has the presence of the force density in the Abraham formulation.

To add to the controversy, from time to time other electromagnetic tensors, such as those of Einstein and Laub [10] and Peierls [11,12], have been constructed. All these tensors exist against the background of standard electromagnetic theory, containing the well-known symmetric, Maxwell and canonical tensors [13]. The Einstein-Laub and Peierls tensors are not formulated from the canonical theory, but directly from the physics of specific material systems, and we leave consideration of these tensors until Sec. II B.

Many of the arguments of the Abraham-Minkowski controversy have centered on features that are either irrelevant for particular experimental conditions, or for optical media in general, or are inconsistent with the fundamental characteristics of the canonical formulation of a tensor from a Lagrangian density. So it is the issues involving the presence of physical boundaries, inhomogeneity, and anisotropy, although important in radiation pressure calculations, ought not, in principle, to arise from a rigorous tensor treatment. For example, the variation of the Lagrangian density used to formulate the tensor assumes that the mathematical boundary is identical to the boundary of the material medium. Indeed, since any electromagnetic tensor is a component in a differential conservation equation, it is not surprising that tensors are indicative of bulk material only, where physical boundaries are removed from the coordinates of interest. Furthermore, the canonical determinations of Abraham [8] and Minkowski [9] stress tensors assume that the material's relative permittivity and permeability are real parameters. Notwithstanding, the literature contains apparent generalizations of these tensors, supposedly obtained by retrospective introduction of material inhomogeneity or anisotropy. Alternatively, one sometimes reads the claim that a particular tensor is unsound because it failed to include these generalizations in the first place [11].

The canonical determination of a stress tensor of a field involves the Hamilton derivative, and it takes place at a level independent of the equations of motion [14,15]. The procedure is to find the conditions for an extremum of action by varying the Lagrangian density with respect to the space-time metric, hence the name "stress tensor." The Hamilton derivative method is applicable to curved and Euclidean space-times, whereas the standard determination of the canonical tensor is restricted to flat space-time. In general, the Hamilton derivative produces a tensor that obeys a conservation law, in that its covariant derivative vanishes. A specialization to Euclidean space-time reduces the covariant derivative to the common form involving the four-vector gradient operator.

A Lagrangian for the electrodynamics of a material medium can be constructed in two distinct forms, depending on whether the phenomenological relative permittivity and permeability of the material are introduced by means of matter-induced currents or by means of an effective modification of the space-time metric. The induced-currents formalism gives rise to a minimal-coupling term in the Lagrangian, and it is perhaps better known than the formalism that involves an effective metric. It also has direct experimental relevance to a material medium at rest on the optical bench. However, the effective-metric formulation is fully covariant and enables a natural introduction of the four-vector velocity of the medium. The effective metric is introduced formally; it is a function of the material four-vector velocity as well as its phenomenological parameters, and it performs the usual metric role of shifting between covariant and contravariant vectors by raising and lowering indices.

After specialization to Euclidean space, the Hamilton derivative of the induced-currents Lagrangian yields

$$\frac{\partial g_S^j}{\partial t} + \sum_{i=1}^{3} \frac{\partial T_S^{ij}}{\partial x^i} = - f_{\mathrm{L}}^j, \tag{2.5}$$

where

$$T_S^{ij} = \varepsilon_0 \left\{ \frac{1}{2}\delta^{ij}\mathbf{E}^2 - E^i E^j \right\} + \mu_0^{-1} \left\{ \frac{1}{2}\delta^{ij}\mathbf{B}^2 - B^i B^j \right\} \tag{2.6}$$

is the symmetric stress tensor which, with a change of sign, is identical to the Maxwell stress tensor. The momentum density is

$$\mathbf{g}_S = \varepsilon_0 \mathbf{E} \times \mathbf{B} \tag{2.7}$$

and $\mathbf{f}_{\mathrm{L}}$ is the Lorentz force density, which for our purposes may be written in the conventional forms

$$\mathbf{f}_{\mathrm{L}} = (\mathbf{P} \cdot \nabla)\mathbf{E} + \frac{\partial \mathbf{P}}{\partial t} \times \mathbf{B} = \varepsilon_0 \chi \left\{ \frac{1}{2} \nabla (E^2) + \frac{\partial}{\partial t} (\mathbf{E} \times \mathbf{B}) \right\}, \tag{2.8}$$

as given by [16]. Note that the momentum density $\mathbf{g}_{\mathrm{S}}$ is the same as $\mathbf{g}_{\mathrm{A}}$ in (2.3) for a nonmagnetic material with $\mathbf{B} = \mu_0 \mathbf{H}$.

The induced-currents Lagrangian contains an inherent matter-field separation; it is therefore unsurprising that it yields the physically significant Lorentz force. This contrasts with the effective-metric Lagrangian, which treats the electromagnetic field and the material medium as a single entity. It is only the effective-metric Lagrangian that canonically produces the Abraham and Minkowski tensors. Therefore, at the center of the Abraham-Minkowski controversy lies a Lagrangian whose division into field and matter components is ambiguous, and perhaps impossible. It is in any case pointless to follow this route as the minimal-coupling induced-currents Lagrangian supplies all that is required for the kind of problem of interest here. To counter the claim sometimes made that only the Abraham tensor is correct, we devote the remainder of this section to an outline of the historical reasons for the claim and to the justification for safely ignoring it. The mathematical detail can be found elsewhere [14,17–19] and is omitted here.

The Hamilton derivative procedure for the effective-metric Lagrangian reveals what has sometimes been seen as a problem, whose apparent resolution is the origin of the claim for the unique correctness of the Abraham form. The variation of the Lagrangian with respect to the effective metric, in fact, produces the Minkowski tensor, with a momentum conservation equation involving the vanishing of its four-divergence, as in the M part of (2.1). The claimed problem with this procedure is that the real space-time metric is not the effective metric but one that, locally at least, is Minkowskian. A correction is supposedly made to ensure that the effective-metric Lagrangian is varied with respect to the Minkowskian metric. This correction results in the introduction of the Abraham tensor, whose divergence vanishes only in combination with the Abraham force, as in the A part of (2.1).

The formalisms involving the induced-currents and effective-metric Lagrangians would be expected to lead to similar results when the space-projection component of the medium's four-velocity vanishes, and this is indeed the case. The Abraham and Minkowski momentum conservation equations describe the same situation and lead to identical results in a spatially stationary frame. This can be appreciated immediately by noting that the effective metric in this case differs from the Minkowski metric only by the presence of the refractive index η in its scalar component. Such an alteration in the speed of the scalar electromagnetic field is simply a gauge transformation and can lead to no new physics. In this situation, the Abraham, Minkowski, and symmetric formulations are equally valid, and (2.1) and (2.5) are equivalent expressions of momentum conservation. In a spatially stationary frame it is meaningless to claim that one or the other tensor is correct. The source of the Abraham-Minkowski controversy lies in a failure to understand fully the canonical construction of the relevant tensors. The difference between the A and M momentum densities in (2.3) can be a source of confusion only if they are detached from the other contributions to the continuity equations in (2.1) and individually given physical significance. The induced-currents formalism, but not, of course, the effective-metric formalism, places no restriction on the material effective permittivity and permeability, as these parameters are implicit within the induced polarization currents. Material inhomogeneity or anisotropy could therefore be introduced in the symmetric (or Maxwell) stress formulation. For consistency, the equivalence between symmetric, Abraham, and Minkowski formalisms for vanishing space-projection component of the four-velocity is restricted to homogeneous and isotropic media.

A possible significance of the Abraham tensor in general relativity, which the Minkowski tensor lacks, is not an issue here. We note, however, that a recent review [20] of 83 years of progress and problems in general relativity and cosmology fails to mention the controversy.

B. Stress tensors and photon drag

Photon drag is a phenomenon of the bulk semiconductor. The light fields vary with only the one coordinate z for the experimental arrangement considered here, with E and B parallel to the x and y axes, respectively, and Maxwell's equations simplify to

$$\frac{\partial E}{\partial z} + \frac{\partial B}{\partial t} = 0 \text{ and } \frac{\partial H}{\partial z} + \frac{\partial D}{\partial t} = 0. \tag{2.9}$$

With material dispersion and loss ignored, the fields are functions of the argument $z-(ct/\eta)$ and the solution $B = \eta E/c$ of (2.9) allows the momentum flux densities from (2.2) and (2.6) to be written

$$T_{\mathrm{A}}^{zz} = T_{\mathrm{M}}^{zz} = \varepsilon_0 \eta^2 E^2 = U \text{ and } T_{\mathrm{S}}^{zz} = \varepsilon_0 (1+\eta^2) E^2, \tag{2.10}$$

where U is the energy density of the electromagnetic field in the bulk semiconductor. The momentum densities from (2.3) and (2.7) have the values

$$g_{\mathrm{A}}^{z} = U/\eta c = g_{S}^{z} \text{ and } g_{\mathrm{M}}^{z} = \eta U/c. \tag{2.11}$$

These are the conventional Abraham and Minkowski momenta and, for consistency with other authors, we follow this nomenclature here to denote the momentum transfers from single photons of energy $\hbar\omega$ to the material medium.

The equivalence of the Abraham, Minkowski, and symmetric formulations is explicitly demonstrated by use of (2.9) to rewrite all three momentum conservation equations from (2.1) and (2.5) as the single form

$$\frac{\partial}{\partial t}\left(\frac{\eta U}{c}\right) + \frac{\partial U}{\partial z} = 0, \tag{2.12}$$

when the force components f_A^z and f_L^z from (2.4) and (2.8) are combined with appropriate terms on the left-hand sides of the relevant equations. The total momentum flux density of the light beam is identical to the energy density U in the semiconductor, and the momentum density is identical to the Minkowski value in (2.11). It may therefore seem that we have simply confirmed the assertion of Gibson *et al.* [7] that the photon drag experiment supports the Minkowski rather than the Abraham formulation, but this is not the case. We have only demonstrated that the Minkowski formulation represents the simplest but not the only valid matter-field division of momentum within a bulk material medium. Provided that the momentum density from (2.3) is not detached from (2.1) and treated in isolation, the Abraham formalism produces exactly the same momentum conservation equation (2.12) as the other approaches.

The Einstein-Laub [10] (E) tensor formalism is also consistent with the photon drag results. The tensor is not constructed canonically by means of a Hamilton derivative but from a consideration of the reaction of a molecular dipole to electromagnetic fields. The momentum conservation equation is

$$\frac{\partial g_{\mathrm{A}}^{j}}{\partial t} + \sum_{i=1}^{3} \frac{\partial T_{\mathrm{E}}^{ij}}{\partial x^i} = -f_{\mathrm{L}}^{j}, \tag{2.13}$$

where

$$T_{\mathrm{E}}^{ij} = \frac{1}{2}\varepsilon_0 \delta^{ij} \mathbf{E}^2 - E^i D^j + \frac{1}{2}\mu_0 \delta^{ij} \mathbf{H}^2 - H^i B^j, \tag{2.14}$$

the momentum density is identical to the Abraham value in (2.3), and the force density has the Lorentz form from (2.8). The momentum conservation equation can again be written in the form of (2.12). The Peierls theory [11,12] uses local, rather than macroscopic, fields with a dielectric function or refractive index that satisfies the Clausius-Mossotti or Lorenz-Lorentz formula. The use of local fields is inappropriate for semiconductors, where the mobile valence and conduction electrons are not localized on individual atoms, and we consider the Peierls theory no further.

The analysis given here provides a somewhat different perspective on the various formulations of the electromagnetic stress tensors to that presented by Brevik [21], but our conclusions are substantially the same. The interpretation of the photon drag experiments is independent of the formalism, whether that of Abraham, Minkowski, or Einstein-Laub. All of these provide valid frameworks for analysis of the measurements, which do not themselves favor one formalism or the other.

III. OPTICAL PROPERTIES OF SEMICONDUCTORS

The optical properties of semiconductors are described in the usual way by a dielectric function $\varepsilon(\omega)$ and a complex refractive index $n(\omega)$, related by

$$\sqrt{\varepsilon(\omega)} = n(\omega) = \eta(\omega) + i\kappa(\omega). \tag{3.1}$$

The photon drag measurements were made at optical wavelengths out to 1.2 mm in Ge and Si. At this wavelength, $\omega\tau \approx 0.6$ in both n- and p-type Ge, and therefore the condition $\omega\tau \ll 1$ for free-carrier absorption independent of the semiconductor band structure was not strictly met. However, the trend toward a limiting value of $\eta_p \hbar\omega/c$, rather than $\hbar\omega/\eta_p c$, for the momentum transfer was clearly apparent. On the other hand, $\omega\tau \approx 0.2$ for n-Si and $\omega\tau \approx 0.1$ for p-Si at 1.2 mm, and the experimental results for this semiconductor were in very close agreement with $\eta_p \hbar\omega/c$ as the limiting value for the momentum transfer at long wavelengths [7,22]. However, it is only for Ge that the optical properties in the submillimeter range have been carefully measured and interpreted [23], and the numerical values quoted below refer to this material.

It can be assumed to good approximations that the charge carriers make a purely imaginary contribution to $\varepsilon(\omega)$, whereas the host material makes a purely real contribution [23] in the regime where $\omega\tau \ll 1$. The phase refractive index has the value $\eta_p \approx 4.0$ for Ge and varies very slowly with frequency for the conditions of the photon drag experiments. The frequency dependence is, however, retained here for greater generality of the calculations that follow. The value of the extinction coefficient $\kappa(\omega)$ is of order 0.1 or less, and we can write

$$\varepsilon(\omega) = 1 + \chi_H(\omega) + \chi_C(\omega) \approx \eta(\omega)^2 + 2i\eta(\omega)\kappa(\omega), \quad \text{with } \kappa(\omega) \ll \eta(\omega), \tag{3.2}$$

where the host and charge-carrier susceptibilities are given by

$$\chi_H(\omega) = \eta(\omega)^2 - 1 \text{ and } \chi_C(\omega) = 2i\eta(\omega)\kappa(\omega). \tag{3.3}$$

Reference [23] provides more detailed justification for the approximations made here.

The thickness D of the semiconductor sample in the photon drag measurements was 30 mm, and the limiting low-frequency power attenuation length was

$$l(\omega) = c/2\omega\kappa(\omega) \approx 2.4 \text{ mm}. \tag{3.4}$$

Thus essentially all of the photon momentum was transferred to the semiconductor. The incident light was in the form of pulses with free-space lengths L of the order of 60 m, with a pulse length L/η_p in Ge of the order of 15 m, much larger than D and l. The experiments were, in fact, done with the sample in open-circuit conditions [5], so that the transfer of momentum to the charges resulted not in a current flow, but in an opposing electric field within the semiconductor. The measured open-circuit voltage across the two ends of the sample was processed to provide values for the momentum transfer per photon.

IV. LORENTZ FORCES ON CHARGE CARRIERS AND HOST

We consider the propagation of a polarized light beam parallel to the z axis with its electric and magnetic fields parallel to the x and y axes, respectively. The Lorentz forces on bulk dielectrics and their surfaces have been calculated quantum mechanically [2,3] on the basis of the susceptibility of the material. We perform a similar calculation of the momentum transfer to the charge carriers based on $\chi_C(\omega)$ from (3.3). It is also shown that, because of the attenuation provided by the charge carriers, there is a further momentum transfer to the host, based on the susceptibility $\chi_H(\omega)$ from (3.3), which was not observable in the photon drag experiments. The charge and host contributions are nicely separated for the two-component optical system used in the photon drag experiments.

The essential features of the momentum transfer are reproduced with the simplifying assumption of an incident light beam of uniform intensity over a cross-sectional area A. The fields within the beam then vary only with z, and the positive-frequency parts of the quantized field operators are [2,3]

$$\hat{E}_x^+(z,t) = i\int_0^\infty d\omega \left(\frac{\hbar\omega}{4\pi\varepsilon_0 c\,\eta(\omega)A}\right)^{1/2} \hat{a}(\omega) \times \exp\left[-i\omega\left(t - \frac{n(\omega)z}{c}\right)\right] \tag{4.1}$$

and

$$\hat{B}_y^+(z,t) = i\int_0^\infty d\omega \left(\frac{\hbar\omega}{4\pi\varepsilon_0 c^3\eta(\omega)A}\right)^{1/2} n(\omega)\hat{a}(\omega) \times \exp\left[-i\omega\left(t - \frac{n(\omega)z}{c}\right)\right]. \tag{4.2}$$

Here $\hat{a}(\omega)$ is the photon destruction operator at frequency ω, with a corresponding creation operator $\hat{a}^\dagger(\omega)$. The operator

$$\hat{n} = \int_0^\infty d\omega \hat{a}^\dagger(\omega)\hat{a}(\omega) \tag{4.3}$$

represents the number of photons in the light beam. The forms of the square-root normalization factors in (4.1) and (4.2) ensure that the time integrated energy flow at $z=0$ is given by (4.3) but with an additional weighting factor $\hbar\omega$ in the integrand [2,3]. Note that, by comparison with a more general field quantization [24], only the real part $\eta(\omega)$ of the complex refractive index is retained in the normalization factors in view of the inequality in (3.2). However, the full refractive index $n(\omega)$ must be kept in the exponentials, as it ensures the proper attenuation of the light beam as momentum is transferred to the charge carriers. The samples used in the measurements [7] were sufficiently long that almost all of the optical momentum was transferred from the light beam, and a subsequent integration over z can accordingly be taken to extend from 0 to ∞.

The rates of change of the momenta of the charge carriers and the host semiconductor, caused by their interaction with the incident light, are determined by integrations of the Lorentz force-density operator over time and over the illuminated spatial region. Only the second term of the first form of the force in (2.8) contributes for the experimental geometry considered here, to give

$$:\hat{\mathbf{f}}(z,t): = :\frac{\partial\hat{\mathbf{P}}(z,t)}{\partial t} \times \hat{\mathbf{B}}(z,t):. \tag{4.4}$$

The first factor on the right is the polarization current, where the operator $\hat{\mathbf{P}}_x^+(z,t)$ is obtained from the electric field operator in (4.1) by insertion of $\varepsilon_0\chi(\omega)$ in the integrand. The explicit forms of the current for the charge carriers and the host are given in (4.6) and (4.10), respectively. The use of normal ordering in (4.4), indicated by the colons, ensures that vacuum contributions are excluded. The time-dependent force operator is obtained by integration over the illuminated sample volume as

$$:\hat{\mathbf{F}}(t): = A\int_0^\infty dz :\hat{\mathbf{f}}(z,t):. \tag{4.5}$$

The force operator vanishes for a transparent sample of infinite length, but nonzero radiation pressure contributions arise in the presence of attenuation and at the surfaces of noninfinite samples [2].

A. Momentum transfer to the charge carriers

The polarization operator is expressed in terms of the electric field operator (4.1) via the charge-carrier susceptibility from (3.3), with the result

$$\frac{\partial \hat{P}_x^+(z,t)}{\partial t} = i\left(\frac{\varepsilon_0 \hbar}{\pi c A}\right)^{1/2} \int_0^\infty \mathrm{d}\omega \omega^{3/2} \eta(\omega)^{1/2} \kappa(\omega) \hat{a}(\omega) \times \exp\left[-i\omega\left(t - \frac{n(\omega)z}{c}\right)\right]. \quad (4.6)$$

The Lorentz force-density operator (4.4), therefore, takes the form

$$:\hat{f}_z^C(z,t): = \frac{\hbar}{2\pi c^2 A} \int_0^\infty \mathrm{d}\omega \int_0^\infty \mathrm{d}\omega' \left(\frac{\omega\omega'}{\eta\eta'}\right)^{1/2} \times (\omega \eta \kappa n' + n^* \omega' \eta' \kappa') \hat{a}^\dagger(\omega) \hat{a}(\omega') \times \exp\left[i(\omega - \omega')t - i(n^*\omega - n'\omega')\frac{z}{c}\right], \quad (4.7)$$

with the convention that the optical variables η, κ, and n are evaluated at frequency ω, while η', κ', and n' are evaluated at ω'. Terms in the products of two creation or two destruction operators are neglected as they do not contribute to the momentum transfer of interest here.

The expectation value of the force density can be calculated for specific states of the incident light, as in previous work [2,3], and this is done in Sec. V. However, for comparison with experiment, we are mainly interested in the total transfer of momentum, represented by the time-integrated force-density operator

$$\int_{-\infty}^{\infty} \mathrm{d}t :\hat{f}_z^C(z,t): = \frac{\hbar}{cA} \int_0^\infty \mathrm{d}\omega \eta_p \omega \hat{a}^\dagger(\omega) \hat{a}(\omega) \exp(-z/l)/l, \quad (4.8)$$

where (3.4) is used to express κ in terms of the power attenuation length l. The derivation above shows that the phase refractive index controls the force density and η is accordingly replaced by η_p in (4.8), which can often be taken as the value at the mean frequency of the incident light. A further integral over the entire illuminated sample gives the form of the operator that represents the total transfer of linear momentum to the charge carriers as

$$A \int_0^\infty \mathrm{d}z \int_{-\infty}^{\infty} \mathrm{d}t :\hat{f}_z^C(z,t): = \int_0^\infty \mathrm{d}\omega \frac{\eta_p \hbar \omega}{c} \hat{a}^\dagger(\omega) \hat{a}(\omega). \quad (4.9)$$

This final expression is the same as the photon-number operator (4.3) but with an additional weighting factor. It shows that the coupling of the light to the charge carriers via the Lorentz force results in a calculated momentum transfer of $\eta_p \hbar \omega / c$ per photon, the same Minkowski value as found in the photon drag measurements [7].

B. Momentum transfer to host semiconductor

The propagation of an optical pulse through a transparent dielectric causes no transfer of momentum to the material, as a positive Lorentz force in the leading part of the pulse is exactly balanced by a negative Lorentz force in its trailing part [2]. However, this balance is removed in the present problem because of the attenuation of the light by its interaction with the charge carriers. This causes the leading part of the pulse at a given time to be weaker than the trailing part and produces a net negative transfer of momentum to the bulk semiconductor. There is also a positive "surface" contribution that arises from the lack of balance between leading and trailing parts as the pulse passes through the integration cutoff at $z=0$ into the active region of the sample at $z>0$. Here we calculate the combined momentum transfer from these two effects and discuss their separate contributions in Sec. V.

The host susceptibility from (3.3) provides a contribution to the polarization operator with the time derivative

$$\frac{\partial \hat{P}_x^+(z,t)}{\partial t} = \left(\frac{\varepsilon_0 \hbar}{4\pi c A}\right)^{1/2} \int_0^\infty \mathrm{d}\omega \omega^{3/2} \eta^{-1/2} (\eta^2 - 1) \hat{a}(\omega) \times \exp\left[-i\omega\left(t - \frac{nz}{c}\right)\right]. \quad (4.10)$$

The Lorentz force-density operator (4.4) is, accordingly,

$$:\hat{f}_z^H(z,t): = \frac{i\hbar}{4\pi c^2 A} \int_0^\infty \mathrm{d}\omega \int_0^\infty \mathrm{d}\omega' \left(\frac{\omega\omega'}{\eta\eta'}\right)^{1/2} [\omega(\eta^2 - 1)n' - n^*\omega'(\eta'^2 - 1)] \hat{a}^\dagger(\omega) \hat{a}(\omega') \times \exp\left[i(\omega - \omega')t - i(n^*\omega - n'\omega')\frac{z}{c}\right]. \quad (4.11)$$

with the same convention as in (4.7) The time-integrated force density is expressed with the use of (3.4) as

$$\int_{-\infty}^{\infty} \mathrm{d}t :\hat{f}_z^H(z,t): = -\frac{\hbar}{2cA} \int_0^\infty \mathrm{d}\omega (\eta_p^2 - 1) \omega \hat{a}^\dagger(\omega) \hat{a}(\omega) \times \exp(-z/l)/\eta_p l, \quad (4.12)$$

where the derivation again shows that the phase refractive index is involved. A further integration over the illuminated sample gives the operator that represents the total transfer of linear momentum to the host semiconductor as

$$A \int_0^\infty \mathrm{d}z \int_{-\infty}^{\infty} \mathrm{d}t :\hat{f}_z^H(z,t): = -\int_0^\infty \mathrm{d}\omega \frac{\eta_p^2 - 1}{2\eta_p} \frac{\hbar \omega}{c} \hat{a}^\dagger(\omega) \hat{a}(\omega), \quad (4.13)$$

a negative quantity in the usual case where $\eta_p > 1$.

The total transfer of momentum to charges and host is represented by the operator

$$A \int_0^\infty \mathrm{d}z \int_{-\infty}^{\infty} \mathrm{d}t :[\hat{f}_z^C(z,t) + \hat{f}_z^H(z,t)]: = \int_0^\infty \mathrm{d}\omega \frac{\eta_p^2 + 1}{2\eta_p} \frac{\hbar \omega}{c} \hat{a}^\dagger(\omega) \hat{a}(\omega), \quad (4.14)$$

where (4.9) and (4.13) are used. The total linear momentum transferred to a general dielectric medium by narrow-band light that carries a single photon of energy $\hbar\omega_0$ in the medium at $z=0$ can be obtained by considerations of energy and momentum conservation. When $\kappa(\omega_0) \ll \eta(\omega_0)$, as assumed here, the result is [3]

$$\text{total momentum transfer} = \frac{\hbar\omega_0}{c}\frac{\eta_p^2+1}{2\eta_p}, \qquad (4.15)$$

consistent with (4.14).

V. TIME-DEPENDENT FORCES FOR SINGLE-PHOTON PULSE

The details of the transfer of momentum to the charge carriers and host crystal can be further investigated with the assumption of incident light in the form of a single-photon pulse. The optical state is defined by [25,26]

$$|1\rangle = \int d\omega \xi(\omega)\hat{a}^\dagger(\omega)|0\rangle, \qquad (5.1)$$

where $|0\rangle$ is the vacuum state, and these states satisfy

$$\hat{a}(\omega)|1\rangle = \xi(\omega)|0\rangle. \qquad (5.2)$$

The normalized function $\xi(\omega)$ describes the spectrum of the photon pulse and a convenient choice is the narrow-band Gaussian of spatial length L,

$$\xi(\omega) = \left(\frac{L^2}{2\pi c^2}\right)^{1/4}\exp\left\{-\frac{L^2(\omega-\omega_0)^2}{4c^2}\right\}, \quad c/L \ll \omega_0. \qquad (5.3)$$

The narrow spectrum ensures that ω can often be set equal to the central frequency ω_0. The peak of the pulse defined in this way passes through the coordinate $z=0$ and into the active region of the sample at time $t=0$.

A. Momentum transfer to the charge carriers

The expectation value of the Lorentz force-density operator (4.7) for the single-photon pulse is obtained straightforwardly with use of the property (5.2) as

$$\langle 1|:\hat{f}_z^C(z,t):|1\rangle = \frac{\hbar}{2\pi c^2 A}\int_0^\infty d\omega\int_0^\infty d\omega'\left(\frac{\omega\omega'}{\eta\eta'}\right)^{1/2} \times(\omega\eta\kappa n' + n^*\omega'\eta'\kappa')\xi^*(\omega)\xi(\omega') \times\exp\left[i(\omega-\omega')t - i(n^*\omega - n'\omega')\frac{z}{c}\right]. \qquad (5.4)$$

The frequency integrals are now to be evaluated without the prior integration over time in (4.8). It is useful to approximate the integrand for frequencies ω and ω' in the vicinity of ω_0. With use of (3.4) and the inequalities in (3.2) and (5.3), standard Taylor expansions give

$$\omega\eta\kappa n' + n^*\omega'\eta'\kappa' \approx 2\omega_0\eta_p^2\kappa(\omega_0) = c\eta_p^2/l \qquad (5.5)$$

and

$$n^*\omega - n'\omega' \approx -i(c/l) + (\omega-\omega')\eta_g, \qquad (5.6)$$

where the group refractive index is defined by

$$\eta_g = \frac{\partial}{\partial\omega}(\omega\eta(\omega))\bigg|_{\omega_0} = \eta_p + \omega_0\frac{\partial\eta(\omega)}{\partial\omega}\bigg|_{\omega_0}. \qquad (5.7)$$

Straightforward integration now gives

$$\langle 1|:\hat{f}_z^C(z,t):|1\rangle = \sqrt{\frac{2}{\pi}}\frac{\eta_p\hbar\omega_0}{AlL}\exp\left\{-\frac{z}{l} - 2\left(t - \frac{\eta_g z}{c}\right)^2\frac{c^2}{L^2}\right\}. \qquad (5.8)$$

The expectation value of the force operator defined in (4.5) is obtained by integration over z and, with use of the standard Gaussian integral and the definition of the complementary error function, the result is

$$\langle 1|:\hat{F}_z^C(t):|1\rangle = \frac{\eta_p\hbar\omega_0}{2\eta_g l}\exp\left(-\frac{ct}{\eta_g l} + \frac{L^2}{8\eta_g^2 l^2}\right) \times\text{erfc}\left(-\frac{\sqrt{2}ct}{L} + \frac{L}{2\sqrt{2}\eta_g l}\right). \qquad (5.9)$$

The force expectation value clearly vanishes in the absence of the attenuation caused by the charge carriers when $l\to\infty$.

Two limiting cases are of interest. For a pulse that is much shorter than the attenuation length, (5.9) reduces approximately to

$$\langle 1|:\hat{F}_z^C(t):|1\rangle = \frac{\eta_p\hbar\omega_0}{2\eta_g l}\exp\left(-\frac{ct}{\eta_g l}\right)\text{erfc}\left(-\frac{\sqrt{2}ct}{L}\right), \quad L/\eta_p \ll l. \qquad (5.10)$$

The value of the complementary error function increases from 0 to 2 over a time of order L/c as the pulse enters the active region of the sample at $z>0$ and the force subsequently decays over a longer time of order $\eta_g l/c$ as the pulse is attenuated. For a pulse that is much longer than the attenuation length, we use the asymptotic form [27]

$$\text{erfc}\, x \to \exp(-x^2)/x\sqrt{\pi} \qquad (5.11)$$

to reduce (5.9) to

$$\langle 1|:\hat{F}_z^C(t):|1\rangle = \sqrt{\frac{2}{\pi}}\frac{\eta_p\hbar\omega_0}{L}\exp\left(-\frac{2c^2t^2}{L^2}\right), \quad L/\eta_p \gg l. \qquad (5.12)$$

The time dependence is now entirely determined by the free-space pulse shape. Note that integration of (5.10) or (5.12) produces a total momentum transfer to the charge carriers of

$$\int_{-\infty}^{\infty} dt\langle 1|:\hat{F}_z^C(t):|1\rangle = \eta_p\hbar\omega_0/c, \qquad (5.13)$$

in agreement with the expectation value of (4.9) for a single-photon narrow-band pulse.

B. Momentum transfer to the host semiconductor

The expectation value of the Lorentz force-density operator (4.11) for the host semiconductor is obtained in a similar manner to that used for the charge carriers. We need the additional Taylor expansion

$$\omega(\eta^2-1)n' - n^*\omega'(\eta'^2-1) \approx i(\eta_p^2-1)(c/l) + (\omega-\omega')(\eta_p^2\eta_g + \eta_g - 2\eta_p), \tag{5.14}$$

and the resulting single-photon pulse expectation value is

$$\langle 1|:\hat{f}_z^H(z,t):|1\rangle = -\sqrt{\frac{2}{\pi}}\frac{\hbar\omega_0}{c\,\eta_p AL}\left\{(\eta_p^2-1)\frac{c}{2l} + (\eta_p^2\eta_g+\eta_g-2\eta_p)\left(t-\frac{\eta_g z}{c}\right)\frac{2c^2}{L^2}\right\} \times\exp\left\{-\frac{z}{l}-2\left(t-\frac{\eta_g z}{c}\right)^2\frac{c^2}{L^2}\right\}, \tag{5.15}$$

analogous to (5.8). The corresponding expectation value of the force operator (4.5) is

$$\langle 1|:\hat{F}_z^H(t):|1\rangle = -\frac{\hbar\omega_0}{2\eta_p\eta_g}\left\{\frac{\eta_p}{l}\left(\eta_p-\frac{1}{\eta_g}\right)\exp\left(-\frac{ct}{\eta_g l}+\frac{L^2}{8\eta_g^2 l^2}\right) \times\operatorname{erfc}\left(-\frac{\sqrt{2}ct}{L}+\frac{L}{2\sqrt{2}\eta_g l}\right) - \sqrt{\frac{2}{\pi}}\frac{1}{L}(\eta_p^2\eta_g+\eta_g-2\eta_p)\exp\left(-\frac{2c^2t^2}{L^2}\right)\right\}, \tag{5.16}$$

analogous to the form of the charge-carrier force in (5.9).

It is again instructive to consider two limiting cases. For a pulse that is much shorter than the attenuation length, (5.16) reduces approximately to

$$\langle 1|:\hat{F}_z^H(t):|1\rangle = -\frac{\hbar\omega_0}{2\eta_p\eta_g}\left\{\frac{\eta_p}{l}\left(\eta_p-\frac{1}{\eta_g}\right)\exp\left(-\frac{ct}{\eta_g l}\right) \times\operatorname{erfc}\left(-\frac{\sqrt{2}ct}{L}\right) - \sqrt{\frac{2}{\pi}}\frac{1}{L}(\eta_p^2\eta_g+\eta_g-2\eta_p) \times\exp\left(-\frac{2c^2t^2}{L^2}\right)\right\}, \quad L/\eta_p \ll l. \tag{5.17}$$

Integration over the time produces a total momentum transfer to the host semiconductor of

$$\int_{-\infty}^{\infty}\mathrm{d}t\langle 1|:\hat{F}_z^H(t):|1\rangle = \frac{\hbar\omega_0}{c}\left\{\left(\frac{1}{\eta_g}-\eta_p\right)+\left(\frac{\eta_p^2+1}{2\eta_p}-\frac{1}{\eta_g}\right)\right\}, \tag{5.18}$$

where the two terms in the main bracket are the contributions of the corresponding terms in (5.17). In the opposite limit of a pulse that is much longer than the attenuation length, use of the asymptotic form (5.11) leads to

$$\langle 1|:\hat{F}_z^H(t):|1\rangle = -\sqrt{\frac{2}{\pi}}\frac{\hbar\omega_0(\eta_p^2-1)}{2\eta_p L}\exp\left(-\frac{2c^2t^2}{L^2}\right), \quad L/\eta_p \gg l, \tag{5.19}$$

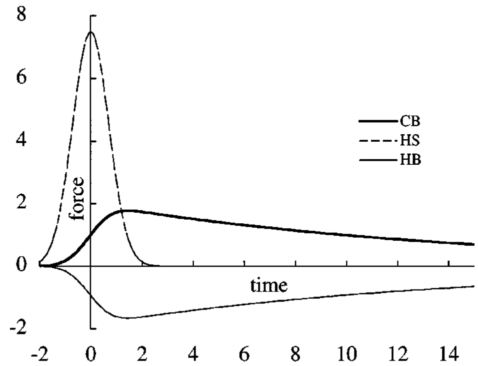

FIG. 1. Time dependence of the forces on charge carriers and host for a semiconductor whose attenuation length is 10 times the single-photon pulse length in the medium. The bulk force on the charges (CB) is given by (5.10), while the bulk (HB) and surface (HS) forces on the host are given by the two terms in (5.17). The force is in units $\hbar\omega_0/2l$, the time is in units $L/\sqrt{2}c$, and $\eta_p=\eta_g=4$.

with a time variation identical to that of the charge-carrier force in (5.12). Further integration of (5.19) over time produces a total momentum transfer of

$$\int_{-\infty}^{\infty}\mathrm{d}t\langle 1|:\hat{F}_z^H(t):|1\rangle = -\frac{\hbar\omega_0(\eta_p^2-1)}{2c\,\eta_p}, \tag{5.20}$$

identical to the summed value in (5.18) and in agreement with the expectation value of (4.13) for a single-photon narrow-band pulse. The total transfer of momentum to the combined system of charge carriers and host semiconductor obtained by summation of (5.13) and (5.20) agrees with (4.15).

C. Discussion

A more complete understanding of the detailed processes of momentum transfer from light to charge carriers and host semiconductor is provided by consideration of some limiting cases.

(i) $L/\eta_p \ll l \ll D$. These conditions ensure that the passage of the pulse into the active region of the sample occurs in a time short compared to the duration of its subsequent attenuation in the bulk material and that none of the light reaches the far boundary of the sample. The surface and bulk contributions to the Lorentz force, described at the beginning of Sec. IV B, are clearly separated in this case. They are characterized by a time dependence similar to that of the Gaussian pulse itself for the surface contribution and an exponential fall-off over the attenuation time $\eta_g l/c$ for the bulk contribution. The time-dependent force (5.10) on the charge carriers shows only a bulk contribution, while the force (5.17) on the host shows both bulk and surface contributions given, respectively, by the two terms in the large bracket. Illustrations of the time dependences of these three contributions are shown in Fig. 1. The total time-integrated force obtained from (5.13) and (5.18) is

$$\int_{-\infty}^{\infty} dt\langle 1|:\hat{F}_z^H(t) + \hat{F}_z^C(t):|1\rangle$$
$$= \frac{\hbar\omega_0}{c}\left\{\underbrace{\frac{\eta_p^2+1}{2\eta_p} - \frac{1}{\eta_g}}_{\text{host surface}} + \underbrace{\left(\frac{1}{\eta_g} - \eta_p\right)}_{\text{host bulk}} + \underbrace{\eta_p}_{\text{charge bulk}}\right\}$$
$$= \frac{\hbar\omega_0}{c}\left\{\underbrace{\frac{\eta_p^2+1}{2\eta_p} - \frac{1}{\eta_g}}_{\text{surface}} + \underbrace{\frac{1}{\eta_g}}_{\text{bulk}}\right\}. \quad (5.21)$$

Combination of the charge carrier and host momentum transfers to the bulk thus produces the Abraham value of $\hbar\omega_0/\eta_g c$, in contrast to the Minkowski value of $\eta_p\hbar\omega_0/c$ produced by the charge carriers alone. This Abraham value represents the total momentum available for transfer from the pulse to the bulk material once it has fully entered the active region of the sample, at the relatively very short time of order L/c. A total bulk momentum transfer of $\hbar\omega_0/\eta c$ was found previously [2] for a semi-infinite dispersionless dielectric in the form of a single-component optical system, without the separation into η and κ contributions, which occurs in the photon drag effect.

(ii) $l \ll D \ll L/\eta_p$. The condition that none of the light reaches the far boundary of the sample is retained, but now the pulse is much longer than both the attenuation length and the sample thickness. This is the regime of the photon drag measurements [7]. The time-dependent forces on the charge carriers and host semiconductor, given by (5.12) and (5.19), respectively, are completely determined by the Gaussian pulse shape. The momentum transfer per photon to the charge carriers retains the Minkowski value of $\eta_p\hbar\omega_0/c$, and the total transfer to the host retains the value given in (5.20), but there is no longer the separation into surface and bulk contributions shown in (5.21). The proportionality of the charge carrier force in (5.12) to the pulse intensity is reproduced in the induced voltage generated across the semiconductor sample by the photon drag effect. The measured voltage forms the basis for the photon drag detector, a device that measures infrared pulse profiles [22].

(iii) $L/\eta_p \ll D \ll l$. Conditions in which the attenuation length is much greater than the sample thickness have been treated in earlier work [2,3,28–30]. This limit is not covered by the present calculations on the photon drag effect, but the results are relevant to the value of the total momentum transfer derived here. With negligible attenuation over the sample thickness D, all of the incident light is either reflected or transmitted. The surface force mechanism described at the beginning of Sec. IV B continues to operate, but the positive force generated as the pulse enters the medium at $z=0$ is at least partially compensated by a negative force generated by the same mechanism as the pulse leaves the medium at $z=D$. The two forces exactly cancel for a slab of dielectric with antireflection coatings [3,29] when the passage of the pulse shifts an initially stationary slab to a new stationary position, with no permanent transfer of momentum. However, application of momentum conservation to the state of the system when the pulse lies within the slab [30] produces an optical momentum of $\hbar\omega_0/\eta_g c$ per photon, in agreement with the Abraham value derived here for the total momentum transfer. More complicated behavior occurs in the presence of reflection at the slab surfaces [2,28]. The results for a transparent slab emphasize the qualitative difference between the reversibility of the surface force and the irreversibility of the bulk force.

VI. CONCLUSIONS

The main results of the calculations are the expressions (4.9) and (4.13) for the transfers of momentum from light to the charge carriers and host semiconductor in the photon drag effect and the expressions (5.9) and (5.16) for the respective time-dependent forces produced by a single-photon pulse. The calculated transfer of linear momentum to the charge carriers in (4.9) agrees with the measurements [7] in assigning the Minkowski value of $\eta_p\hbar\omega/c$ per photon. The transfer of momentum to the host semiconductor in (4.13) is essentially fixed by momentum conservation, which requires the total momentum transfer per photon to have the value given in (4.15). The momentum transfers calculated in Sec. IV are valid for all forms of incident light that satisfy the overall condition $\omega\tau \ll 1$ discussed in Sec. III.

Additional information on the detailed processes of momentum transfer is provided by the calculations of Sec. V for an incident single-photon pulse, and the significance of the results is discussed in Sec. V C. The main additional feature emerges for pulses much shorter than the attenuation length, when the momentum transfer to the host semiconductor can be separated into surface and bulk contributions, as in (5.21); the total bulk momentum transfer in this case, charges plus host, equals the Abraham value of $\hbar\omega_0/\eta_g c$. The practical device of the photon drag detector relies on the opposite limit of pulses much longer than the attenuation length, when the time-dependent voltage generated by the forces on the charge carriers mimics the optical pulse shape.

Our method of calculation, based on evaluation of the Lorentz force, has the advantage of providing results for the momentum transfers from light to macroscopic media measurable in experiments. A great deal of previous work, well reviewed in [1], is concerned with the identification of the momentum carried by the photon in dielectric media, often viewed as a conflict between the Abraham and Minkowski expressions, with one or the other or neither regarded as the "correct" answer for the system in question. We have argued in Sec. II that there is no conflict between the Abraham, Minkowski, and other formulations of the electromagnetic stress tensor. Furthermore, we have shown in Secs. IV and V that there is no unique expression for the momentum transfer from light to matter in the photon drag effect but that the Abraham and Minkowski values can both, in principle, be observed by appropriate measurements. Another focus of previous work that is untouched by our method of calculation is the devision of total momentum between "electromagnetic" and "material" contributions, and we can make no comment on this aspect of the problem.

Our specific results for the photon drag system harmonize with some previous, more general discussions of photon momentum. Thus, Gordon [16] finds the Abraham form

$\hbar\omega_0/\eta_g c$ for the "true field momentum" and the Minkowski form $\eta_p\hbar\omega/c$, identified as the "pseudomomentum," for the momentum transfer to material objects in dielectric media. Joyce [31] presents a classical particle description of the radiation pressure problem, with the Abraham form for the "energy-carrying momentum" that determines the displacement of a slab and the Minkowski form for the "impulsive momentum" that determines the momentum transfer to a reflector immersed in the medium. McIntyre [32] considers more general forms of wave and matter, partly in the context of fluid mechanics, and reaches conclusions similar to those in [16]. Nelson [33] finds the Abraham form for the electromagnetic field in nonmagnetic media and the Minkowski form for the "wave momentum" that enters wave-vector conservation relations. Very recently, Garrison and Chiao [34] have given a quantum theory of the electromagnetic momentum in a dispersive dielectric, finding that the Abraham form determines the rigid acceleration of a dielectric, while their "canonical momentum," equivalent to the Minkowski form as defined by us, determines the transfer to immersed objects. These authors also use macroscopic field quantization of the classical expression to derive a form of "Minkowski momentum" that differs from ours by an additional factor η_p/η_g, not detectable in the photon drag experiments of [7]. By contrast, Mansuripur [35,36] has considered experiments sensitive to the total momentum transfer (4.15), equal to the arithmetic mean of the Abraham and Minkowski forms.

The best measurements of the momentum transfer to an immersed object are those of Jones and Leslie [37]. They suspended a highly reflecting mirror in a range of dielectric liquids and observed the Minkowski transfer of $\eta_p\hbar\omega/c$ per photon. The measurements were sufficiently accurate to establish the occurrence of the phase refractive index η_p in the momentum transfer, and not the group value η_g. The system is somewhat similar to that in the photon drag effect, with the optical properties of the mirror determined by its extinction coefficient κ and those of the host liquid by its real refractive index η_p. A Lorentz force calculation [2] again provides an expression for the momentum transfer in agreement with experiment. It was shown that the liquid takes up a momentum transfer equal to the difference between the Abraham and Minkowski values, as given by the "host bulk" term in (5.21), although the theory in [2] did not distinguish phase and group refractive indices. A similar interpretation of the mirror experiments has been given by Mansuripur [35]. On the basis of these two examples, it appears that the total momentum transfer to bulk material, free of any boundary or surface effects, has the Abraham value of $\hbar\omega_0/\eta_g c$ but that the transfer to an attenuating subsystem within the bulk material has the Minkowski value of $\eta_p\hbar\omega/c$.

The photon drag system treated here has a remarkable combination of properties, in providing bases for both the practical device of the photon drag detector and the theoretical understanding of momentum transfer from light to matter. The separate contributions of the two system components to the real and imaginary parts of the complex refractive index in the photon drag effect provide a uniquely simple division of the momentum transfers to the host semiconductor (unmeasured) and charge carriers (measured), respectively.

Note added in proof: The Minkowski momentum transfer to the charge carriers in the photon drag effect has been derived by an alternative treatment [38].

ACKNOWLEDGMENTS

We thank Andy Walker for encouragement and Maurice Kimmitt for much advice on the procedures and interpretation of the photon drag experiments. Work at the University of Strathclyde was supported by the UK Engineering and Physical Sciences Research Council.

[1] I. Brevik, Phys. Rep. **52**, 133 (1979).
[2] R. Loudon, J. Mod. Opt. **49**, 821 (2002).
[3] R. Loudon, Phys. Rev. A **68**, 013806 (2003).
[4] A. M. Danishevskii, A. A. Kastalskii, S. M. Ryvkin, and I. D. Yaroshetskii, Sov. Phys. JETP **31**, 292 (1970).
[5] A. F. Gibson, M. F. Kimmitt, and A. C. Walker, Appl. Phys. Lett. **17**, 75 (1970).
[6] A. F. Gibson and S. Montasser, J. Phys. C **8**, 3147 (1975).
[7] A. F. Gibson, M. F. Kimmitt, A. O. Koohian, D. E. Evans, and G. F. D. Levy, Proc. R. Soc. London, Ser. A **370**, 303 (1980).
[8] M. Abraham, Rend. Circ. Mat. Palermo **28**, 1 (1909); **30**, 33 (1910).
[9] H. Minkowski, Nachr. Ges. Wiss. Goettingen, Math.-Phys. Kl., 53 (1908); Math. Ann. **68**, 472 (1910).
[10] A. Einstein and J. Laub, Ann. Phys. **26**, 532 (1908); **26**, 541 (1908).
[11] R. Peierls, Proc. R. Soc. London, Ser. A **347**, 475 (1976); **355**, 141 (1977).
[12] R. Peierls, *More Surprises in Theoretical Physics* (Princeton University Press, Princeton, 1991) pp. 37–42.
[13] J. D. Jackson, *Classical Electrodynamics*, 3rd ed. (Wiley, New York, 1999).
[14] A. S. Eddington, *The Mathematical Theory of Relativity*, 2nd ed. (Cambridge University Press, Cambridge, England 1924).
[15] L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields*, 4th ed. (Pergamon Press, Oxford, 1975).
[16] J. P. Gordon, Phys. Rev. A **8**, 14 (1973).
[17] W. Pauli, *Theory of Relativity* (Pergamon Press, Oxford, 1958).
[18] S. Antoci and L. Mihich, Nuovo Cimento Soc. Ital. Fis., B **112**, 991 (1997); Eur. Phys. J. D **3**, 205 (1998).
[19] U. Leonhardt, Phys. Rev. A **62**, 012111 (2000).
[20] G. F. R. Ellis, Class. Quantum Grav. **16**, A37 (1999).
[21] I. Brevik, Phys. Rev. B **33**, 1058 (1986).
[22] A. F. Gibson and M. F. Kimmitt, in *Infrared and Millimeter Waves*, edited by K. J. Button (Academic Press, New York, 1980), Vol. 3, p. 181.
[23] J. R. Birch, C. C. Bradley, and M. F. Kimmitt, Infrared Phys. **14**, 189 (1974).
[24] R. Matloob, R. Loudon, S. M. Barnett, and J. Jeffers, Phys. Rev. A **52**, 4823 (1995).

[25] K. J. Blow, R. Loudon. S. J. D. Phoenix, and T. J. Shepherd, Phys. Rev. A **42**, 4102 (1990).
[26] R. Loudon, *The Quantum Theory of Light*, 3rd ed. (Oxford University Press, Oxford, 2000).
[27] H. B. Dwight, *Tables of Integrals and other Mathematical Data*, 4th ed. (Macmillan, New York, 1961) §592.
[28] M. S. Kim, L. Allen, and R. Loudon, Phys. Rev. A **50**, 3614 (1994) (see also the Appendix of [2]).
[29] M. Padgett, S. M. Barnett, and R. Loudon, J. Mod. Opt. **50**, 1555 (2003).
[30] R. Loudon, Fortschr. Phys. **52**, 1134 (2004).
[31] W. B. Joyce, Am. J. Phys. **43**, 245 (1975).
[32] M. E. McIntyre, J. Fluid Mech. **106**, 331 (1981).
[33] D. F. Nelson, Phys. Rev. A **44**. 3985 (1991); R. Loudon, L. Allen, and D. F. Nelson, *ibid.* **55**, 1071 (1997).
[34] J. C. Garrison and R. Y. Chiao, Phys. Rev. A **70**, 053826 (2004).
[35] M. Mansuripur, Opt. Express **12**, 5375 (2004).
[36] M. Mansuripur, A. R. Zakharian and J. M. Moloney, Opt. Express **13**, 2064 (2005).
[37] R. V. Jones and B. Leslie, Proc. R. Soc. London, Ser. A **360**, 347 (1978).
[38] M. Mansuripur, Opt. Express **13**, 2245 (2005).

Colin-Baxter.com

End of Document