Case for Support:

# Quantum Mechanics of Atoms Confined to Two-Dimensional Motion

## A. Introduction/Abstract

A rigorous treatment of the centre-of-mass dynamics for a moving atom in an electromagnetic field reveals the existence of the Röntgen interaction. This term has been identified as the responsible agent for several areas of important new physics in the fields of laser cooling and spectroscopy, as well as providing fundamental insights into the general theory of radiation-matter interaction. In particular, the Röntgen energy, which is linear in the atomic velocity, may be made to assume a form that is formally analogous to a Chern-Simons interaction in one temporal and two spatial (1+2) dimensions. This occurs when the atom is confined to a two-dimensional plane by means of a particular arrangement of static electric and magnetic fields. One important and exciting consequence of the theory is that under certain circumstances the atom's orbital angular momentum spectrum is one of half-integer multiples of $\hbar$.

We propose to investigate the gross motion effects of such atoms, exploiting our extensive skill-base in this area. Transitions between angular momentum states may be induced by means of a suitable probe beam, and we will carry out a full enumeration of the radiation-induced changes in the atomic orbital angular momentum operator and states. The present proposal is for funds to carry out a research programme over a period of two years.

Chern-Simons theories describe the behaviour of classical and quantum fields in reduced-dimensional spacetimes. They are the subjects of much world-wide research. In part, this has been prompted by their remarkable properties: the appearances of fractional spin statistics and novel soliton solutions; unexpected links with two-dimension conformal field theory and knot theory; etc.

Our proposed research, therefore, will link an abstract and general theory with the real and utilitarian world of electromagnetic interaction. It will not only have the depth to provide a detailed understanding of atomic gross motion effects in (1+2) dimensions but also be sufficiently broad so as to be applicable to other confined systems – such as ions.

We also propose to calculate the parameters which are necessary to determine the feasibility of experimentally seeing the Röntgen-induced Chern-Simons effects. This will complement an early expression of interest shown by an experimental group at the University of Maryland in devising suitable experiments.

## B. Scientific/Technological Relevance

Interest in the angular momentum properties of laser cooled atoms and ions is burgeoning. Advances in laser technology have motivated and spurred the study of the orbital angular momentum properties of light beams – particularly those beams which propagate according to the paraxial approximation, such as Laguerre-Gaussian modes [1]. There is now intense effort to determine how such beams might be used in the generation of atomic orbital angular momentum states, and their exploitation in laser cooling and trapping [2]. Also, efforts are underway to build so-called "optical spanners" for the use in a number of instruments, such as, for example, confocal microscopes. These will use specially prepared laser beams to impart angular momentum to micron-size particles. Indeed, the first direct observation of the transfer of angular momentum from a laser beam to such particles has only just been reported [3].

One particularly innovative recent development is the revelation that the angular momentum spectrum of a cold Rydberg atom may be made to assume half integer-multiples of $\hbar$ [4]. This novel feature comes about when the atom's Röntgen interaction takes on a form of a Chern-Simons topological term; a condition that is achieved by confining the atom to a plane in a particular arrangement of crossed static electric and magnetic fields. This work has stimulated worldwide interest [5].

In the last two or three years, the Röntgen interaction has revealed much interesting new physics in the areas of spectroscopy and quantum optics. For example, it makes novel contributions to atomic gross motions [6]-[8] and it is responsible for introducing a quantum phase in a dipole moving in a constant magnetic field [9]. Similarly in recent years, the study of field theories in spaces of reduced dimensions has received considerable attention [10]. Such theories are used in descriptions of the quantum Hall effect and high $T_c$ superconductivity [11]. They are now beginning to find important applications in nonlinear quantum optics [12]. Hence, the identification of a form of the Chern-Simons interaction at the atomic level is an exciting development. Our proposed research will capitalise on this development, and will allow, for the first time, direct explorations to be made of the rôle of Chern-Simons theory in predicting novel physics within the settings of odd-dimensional spacetimes.

The recent observation of the Bose-Einstein condensation in a dilute atomic vapour [13] typifies the hectic pace at which atoms have been cooled to lower and lower temperatures. We have now reached a stage where the unique properties of cooled atoms and ions have a great potential for technological exploitation. A realistic implementation of quantum computing is one of the many exciting possibilities being discussed in the literature [14]. Formally, a half-integer angular momentum spectrum comes about when the angular momentum eigenstates are constructed from a dyadic product of a set of number states and a set of states representing the orientations of the configuration-space axes. A unitary operator may be formed from these eigenstates which has the same behaviour as a controlled-NOT gate. Such a situation suggests the intriguing possibility that the orbital angular momentum properties of cold Rydberg atoms may be usefully implemented in a quantum computer.

## C. Relevance to Beneficiaries

The proposed research involves fundamental physics, and, although it has no immediate industrial application, it will significantly contribute to our understanding of atomic angular momentum effects. This is crucial if these effects are to be technologically exploited.

The immediate impact of the proposed work will be felt by fellow researchers in quantum optics and laser physics at home and abroad. As indicated in Section B, there is increasing interest in the orbital angular momentum properties of light, which is part of an intense international activity in the study and exploitation of the mechanical effects of radiation. Hence, the proposed research is highly topical. Its novelty and innovation has already stimulated efforts outside the United Kingdom [5]. Indeed, an experimental group at the University of Maryland have expressed a serious interest in looking for Röntgen-induced Chern-Simons effects. This initiative is at a tentative stage of development and it is therefore vital not to loose the opportunity to produce work of major international importance.

## D. Dissemination and Exploitation

The University of Essex has an energetic and versatile quantum optics research group, consisting of Professor R. Loudon, Professor L. Allen, Professor M. Babiker, Dr. M. Artoni and Dr. C. Baxter. Together, we have considerable expertise and experience in the field of radiation-induced mechanical effects, and, over the last two years, we have been responsible for over eight research publications in this specific area – all of which have appeared in the leading refereed journals, including two in *Physical Review Letters*. We are part of a quantum optics European Community research network. We also have research links with a number of laboratories and universities, including Imperial College (Optics Section), the Universities of Strathclyde and Leiden, British Telecom Research Laboratories (Martlesham) and the Defence Research Agency (Malvern). Therefore, the research experiences gained from the proposed work will be quickly disseminated throughout a large community, with an inevitable and beneficial cross-fertilisation of ideas.

Recent publications by Professor R. Loudon and Dr. C. Baxter have appeared in *Nature*, *Physical Review Letters*, *Physical Review*, *Optics Letters*, *Optics Communications* and *Annals of Physics*. Also, articles for the general reader have been written by the same authors for such magazines as *New Scientist* and *Physics World*. Thus, it is our intention that the results of the proposed research will appear in one or more of the above refereed journals, as well as been given directly to the beneficiaries through seminar and conference presentations (see Section F). We also intend to write from time to time appropriate articles for the non-specialist.

Although it is unlikely that any patent or intellectual property rights would accrue directly from the proposed research, the University of Essex, should the need arise, has appropriate personnel and administration procedures dedicated to the protection and exploitation of such rights.

## E. The Programme

### 1. Summary of the Background Physics

Chern-Simons theory was developed from a study of vector and tensor gauge-fields in (1+2) dimensions. The motivation drew on an apparent connection of such fields to the high temperature behaviour of four-dimensional models [15], leading, it was thought, to a greater awareness of the problem of reconciling quantum theory with general relativity. Although it is true that quantum gravity can be solved exactly in (1+2) dimensions, some of the earlier optimism has proved to be premature. For example, the hope that the metric of four-dimensional spacetime would emerge under certain conditions as a nonzero vacuum expectation value remains unrealised [16]. To be sure, the mathematical richness provided by the exact solutions of some of the Chern-Simons models can still bring a rewarding physical insight to theories in four-dimensional spacetime, but, in recent years, a move has been made to new areas of application. In particular, it has been found that a direct exploitation of the Chern-Simons formalism often yields effective field theories for important real systems, such as strings, fibres, membranes and surfaces, where one or two spatial dimensions are either absent from the theory or do not play a significant rôle. Thus it is, these days, that the quantum Hall effect and high $T_c$ superconductivity – two planar phenomena – are described in terms of Chern-Simons theory, which is viewed as the field theoretical relative of Landau's work on charged particles in an external magnetic field, and of the Aharonov-Bohm effect [17].

In essence, the Chern-Simons term is a gauge-dependent energy in a Lagrangian for a system defined in an odd number of spacetime dimensions, which leaves the equations of motion gauge invariant. This is easily demonstrated in the case of electrodynamics in (1+2) dimensions. Here, the Chern-Simons term is proportional to $\epsilon^{\mu\nu\eta}F_{\mu\nu}A_\eta$, where $\epsilon^{\mu\nu\eta}$ is the three-dimensional permutation tensor and the other symbols have their usual meanings. In the Coulomb gauge, this reduces to $\epsilon^{ii'}\dot{A}_i A_{i'}$. Retaining the covariant notation, with the metric $(+,-,-)$, the simplest particle-analogy may be described by the Lagrangian

$$L(q,\dot{q}) = -(1/2)\dot{q}_i\dot{q}^i - \epsilon^{ii'}\dot{q}_i q_{i'} + (1/2)q_i q^i, \tag{1}$$

where $\epsilon^{ii'}\dot{q}_i q_{i'}$ is the formally equivalent Chern-Simons term. Equation (1) is invariant under $U(1)$ gauge transformations $q^1 \to q^1\cos\phi - q^2\sin\phi$, $q^2 \to q^1\sin\phi + q^2\cos\phi$, and represents a two-dimensional oscillator whose frequency, mass and Chern-Simons strength terms have been normalised.

The orbital angular momentum spectrum of a planar system defined by the Lagrangian (1) is half-integer in the so-called "pure Chern-Simons theory" – that is, when the kinetic energy term $-(1/2)\dot{q}_i\dot{q}^i$ becomes negligible. It is sometimes thought that half-integral orbital angular momentum cannot occur in nature. However, the usual argument, which rests on the impossibility of a particle's wavefunction being anything other than single-valued [18], is inappropriate in this case since a portion of space is denied to the particle [19].

We have recently shown [4] that such a system can in principle be realised by confining a cold Rydberg atom to a plane by means of static electric and magnetic fields. By including the atom's Röntgen energy, the Lagrangian formally takes on the appearance of (1) and the resulting Chern-Simons effects arise naturally from the physics. This is a unique development since the Chern-Simons interaction has previously either been introduced into the Lagrangian as a fictitious gauge potential, as is the case for anyons, or arose from the use of an effective Lagrangian. We are now in the position for the first time of being able in principle to realise on the atomic level the Chern-Simons' feature of fractional angular momentum.

The Röntgen interaction is only apparent in a canonical treatment of the centre-of-mass dynamics of an aggregate of charges in the presence of electric $\mathbf{E}$ and magnetic $\mathbf{B}$ fields [6]. It is necessary in order to conserve momentum and to ensure gauge invariance. Fundamentally, the interaction is a characteristic of the convection current density, resulting from the charge aggregate's gross motion [20]. This current is extra to the aggregate's internal (bound) currents and to any free currents present due to a nonzero overall charge. If the charge aggregate is of atomic dimensions and has a gross canonical momentum $\mathbf{P}$ in three-dimensional space, the Röntgen interaction reduces to $\mathbf{P}\cdot\mathbf{d}\times\mathbf{B}$ in the approximation (dipole) of writing the polarisation to the first order in the dipole moment $\mathbf{d}$. This takes on the appearance of a Chern-Simons interaction if the dipole is confined to a plane by a particular arrangement of static $\mathbf{E}$ and $\mathbf{B}$ fields. The pure Chern-Simons theory is achieved in this case by ensuring that the Röntgen interaction dominates – a condition which implies that the atom has a negligible kinetic energy and is prepared in an internal Rydberg state. Recent work by members of the Essex group (see Section D) suggests that a suitable environment in which to prepare these conditions might be created by means of counter-propagating Laguerre-Gaussian beams [21].

**2. Plan of Work**

The immediate aims of the project are given below in items (i) to (viii). It will be appreciated that many of these steps are inter-related and the list does not imply a set of isolated problems; however, they do form appropriate criteria by which one might measure the success of our proposed research programme.

**i. The Dynamical Theory**

General calculations will be made of the orbital angular momentum dynamics for an atom confined to (1+2) dimensions. Attention will be given to the identification of those atomic and radiation states of the system that result in the predominance of the Röntgen energy, precipitating a shift to pure Chern-Simons theory and a half-integer angular momentum spectrum.

**ii. The Nature of the Probe Beam**

In order to stimulate atomic orbital angular momentum transitions a suitable radiation beam may be used as a probe. We will seek to answer the question as to whether such a suitable probe must of necessity contain an inherent orbital angular momentum component, such as that of a Laguerre-Gaussian beam. This problem hinges on the degree in which the atomic dipole moment influences the atomic orbital angular momentum dynamics. This is a non-trivial issue because the dipole moment couples directly to the probe via the usual dipole interaction. Indeed, exploratory calculations suggest that an azimuthal Doppler shift may perhaps be induced, in the motion of an atom confined to two dimensions, by a single plane-wave radiation beam [22].

**iii. The Nature of the States**

An attempt will be made to understand the nature of the atomic orbital angular momentum states in (1+2) dimensions. Theoretically, one may write these as coherent states: more precisely, as eigen states of an annihilation operator formed from a non-Hermitian combination of the conjugate co-ordinates [23]. There is a reduction in the number of conjugate variables in the shift to pure Chern-Simons theory. We will seek to clarify the significance of this reduction, in relation to the physical states of the system.

**iv. Relevance to Other Bound System**

Since we propose to develop a general theory, which will be applicable to ions as well as atoms, we will extend our calculations to cover the case where the confined bound-system possesses an overall net charge.

**v. The Experimental Component**

It is our intention to conduct investigations into the circumstances and conditions under which Röntgen-induced Chern-Simons effects might be observed under realistic experimental conditions. One immediate end-user of this work is likely to be the experimental group at the University of Maryland, who, as described in Section C, have expressed a strong interest in looking for such effects.

**vi. Collective Effects**

It is anticipated that if a large number of cold Rydberg atoms were confined to (1+2) dimensions, collective properties, such as those induced by the overlap of the individual atomic thermal wave-functions, would greatly affect the orbital angular momentum dynamics. This is an exciting and very topical problem, in view of the recent achievement in obtaining a Bose-Einstein condensation in an atomic vapour [13]. We will seek to clarify the nature of the physics, which is induced by collective effects. Since the region of very cold temperatures is precisely the region at which the pure Chern-Simons theory might apply, it is not clear, at this stage, what form any condensation would assume in (1+2) dimensions. In general, we anticipate that numerical calculations will be necessary in order to stimulate variations in the key parameters, and to solve the coupled differential equations governing the real-time evolution of the interaction. In Section F, we have made a request for funds to meet the costs of appropriate computer facilities.

**vii. The Peierls' Substitution**

If the Röntgen interaction is put into a Chern-Simons' form, the Hamiltonian takes on an appearance similar to that for a charged particle in a magnetic field. We know that the presence of an external potential affects the energy-degenerate states in the lowest Landau level of such charged particles. It is also well known that the solution of this problem is obtained by means of the so-called Peierls' Substitution [24]. The result, in the approximation of a strong magnetic field and weak potential, is that the energy eigenvalues are changed by an amount that is numerically equal to the eigen energy of a potential written in terms of canonically conjugate variables. We intend to investigate whether an analogous effect takes place in the case of an atomic dipole confined to a two-dimensional space. It is likely that such an effect will introduce new physics into the area of laser cooling.

**viii. Concluding Work**

The results of the work outlined in (i)-(vii) will be examined in the context of general Chern-Simons theory by comparing them to those obtained in analogous situations for fields confined to $(1+2)$ dimensions. We will also tentatively examine the apparent formation of a controlled-NOT gate from orbital angular momentum product states, as mentioned in Section B. This will enable us to judge the feasibility of more extensive studies.

It is anticipated that the "benchmarks" (i), (ii), (iii) and (iv) will be attained by the end of the first year of our proposed two-year research programme; the remaining items, that is (v), (vi), (vii) and (viii) by the end of the second year. This is illustrated in diagrammatic form in *Case for Support*, Part 3.

The programme of work, we have outlined in this section, is a consequence of having already carried out a pilot study [4]; one that has enriched our overview of the general issues involved in engaging problems in Chern-Simons' theory. We are therefore confident that the proposed programme will form a firm foundation for the practical understanding atomic gross motion effects in (1+2) dimensions, as well as making a significant contribution to the search for physical meaning to Chern-Simons theories.

## F. Management and Resources

The complexity of the research warrants the appointment of a full-time research assistant, who will carry out the above programme of research. We propose to employ Dr. C. Baxter for this purpose. His particular experience in quantum optics qualifies him as the unique candidate. We therefore request funds in order to employ Dr. C. Baxter for a period of two years, beginning at salary point 10.

The progress of the work will be reviewed through weekly meetings between Dr. C. Baxter and Professor R. Loudon. The management and control of the programme will rest with Professor R. Loudon.

We also request the sum of £2300 for computer facilities. This will be used to purchase:- (1) a suitable PC and software for connection to the University of Essex Computer Services' DEC alpha servers; (2) a dedicated postscript laser printer for the preparation of materials for publication, etc. The details of this computer-equipment purchase are given in Section 16 of EPS (RP).

We further request the sum of £3500 to meet the costs of essential travel. Again, this is detailed in EPS (RP), Section 13. We anticipate that this will include one trip to the University of Maryland to discuss with colleagues (see Section C), and attendance at two major international conferences, in order to make formal presentations of the results of our proposed research. Attendance at such conferences, which, of course, enhances the research profile of the United Kingdom, is an essential pivot in the dissemination of our work and will consolidate its international awareness. To maximise the scope for dissemination, we intend to participate in a broad-based meeting covering all aspects of quantum electronics and laser science, as well as one dedicated to the fundamental theory of quantum optics. We anticipate that the CLEO/QELS Conference, Baltimore, 1997, and the European Quantum Optics Conference, Italy, 1998 will form the appropriate and respective choices.

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