

Case for Support:

# Quantum Theory of Radiation Pressure in Material Bodies

C. Baxter, S. M. Barnett

Department of Physics and Applied Physics,  
University of Strathclyde, Glasgow G4 0NG.

## A. Introduction

Despite the enormous success of quantum optics in the last thirty-years, there is still no fully-quantised theoretical description of the mechanical effects of radiation. This deficiency has impeded the development of the mechanical properties of novel light-sources. Thus, for example, despite knowing for some time that radiation pressure fluctuations limit the ultimate resolving power of optical interferometers [1], the use of non-classical light in improving the performance in high-precision metrology remains largely unexploited.

The interferometric detection of gravity waves is one area that would benefit from a thorough understanding of radiation pressure. Another area — one which is strategically important — is nanoscale engineering. Here, great commercial potential lies in utilizing light in the transport of quantum particles. Although electromagnetic momentum is weak, mechanical excitation on the nanometre scale is possible by the use of modern high-intensity sources of laser light. A recent suggestion is an optomechanical effect that might form the basis of an opto-mechanical transducer [2]. Nanoscale engineering would quickly exploit the advantages of a consistent theory of radiation pressure.

We therefore propose to develop a quantum theory of radiation pressure from first principles, exploiting our extensive experience in this area. We take the view that the mechanical properties of light can only be reliably investigated in terms of a theory where the electromagnetic field is formally quantised. The research will be based on single-photon pulses, and cover the propagation of light through dispersive and absorbing dielectrics. This will allow specialisations to be made to configurations that are directly experimentally relevant: where the light is coherent and the optics involve mirrors, and materials which are effectively transparent.

The general theory will enable the nature of surface forces to be determined. This is a tantalising prospect. For the first time, the significance of the early work of Planck [3] and Poynting [4] will be evaluated according to the tenets of quantum mechanics. In addition, one will be able to adjudicate the long-running but yet unresolved issue of the correct form assumed by the radiation momentum density by assessing the physical meaning of the momentum expressions proposed by Abraham, Minkowski, Peierls, etc. [5].

## B. Scientific and Technological Relevance

The proposed research will strongly impact on the burgeoning field of quantum-optical processes in media. It will define and elucidate fundamental issues in this area, involving radiation pressure and the influence of material interfaces. In particular:—

- How is the pressure related to various expressions for the electromagnetic momentum density?
- What are the magnitude and direction of the force at the interface?
- What information does the magnitude of the force at an interface, such as a mirror, immersed in a dielectric liquid provide on the electromagnetic momentum density in the liquid?

These are important questions: they are unresolved and have been topics of intense international debate for many years.

The research will also impact on related physics involving Casimir forces in various systems [6]; the phenomenon of sonoluminescence [7]; and the different kinds of electromagnetic momenta in infinite dielectrics

[8]. These studies complement the present proposal, which builds on previous work on radiation pressure in dielectric slabs [9] and mirrors [10].

The proposed research is timely since the basis of the necessary formalism for the states of the light [11] and the electromagnetic field quantisation in dielectric media [12] has only recently been determined. Its promise in terms of advancing fundamental understanding of radiation pressure is detailed in other sections of this Case for Support. It is now time for a proper understanding of the limits imposed by quantum mechanics on the mechanical effects of radiation.

### **C. Relevance to Beneficiaries**

The proposed research involves fundamental physics. It will significantly contribute to an understanding of the quantum processes involved in radiation pressure. This is crucial if these underlying processes are to be technologically exploited in, for example, those areas outlined in Section A.

The thrust of the proposed work will be felt by fellow researchers in quantum optics and electrodynamics. As indicated in Section A, there is increasing need for a quantum theory of radiation pressure to enable the exploitation of the mechanical effects of radiation. We believe the proposed research is ideally suited for this purpose, and will provide synergy in the current climate of intense international activity.

The proposers have research contacts with some of the main contributors in the field. These include:–

- (1) R. Loudon (Essex); fundamental aspects of radiation pressure.
- (2) M. Babiker (Essex); fundamental aspects of radiation pressure.
- (3) G. Barton and C. Eberlein (Sussex); Casimir forces; quantum electrodynamics of moving interfaces.
- (4) G. Leuchs (Erlangen, Germany); gravitational wave detection.
- (5) J. Hough (Glasgow); gravitational wave detection.
- (6) A. Heidmann (Paris); quantum-limited interferometry.
- (7) P. W. Milonni (Los Alamos); fundamental aspects of optical theory.

The work of (3), on vacuum radiation pressure at interfaces, is complementary to the research proposed here. It is intended to strengthen the existing contacts by visits, as detailed in Section F. As mentioned in Section D below, we would be able to exploit the resources of the Research and Consultancy Services of the University of Strathclyde in order to formalise any research partnership, should such an arrangement, at a later date, be determined to be of advantage.

### **D. Dissemination and Exploitation**

The University of Strathclyde has an energetic and versatile activity in quantum optics including both theoretical and experimental work. There is also considerable expertise and experience in the field of quantized fields in dielectric and other media and of radiation-induced mechanical effects, especially in the area of laser cooling. In the last five years, the applicants have written some twenty papers relevant to these areas, all of which have appeared in the leading refereed journals. We are the co-ordinating group for the European Community TMR Quantum Structures. There are also research links with a number of laboratories and universities, including Universite Pierre et Marie Curie and Ecole Normale Superieure in Paris, Max Planck Institut fur Quantenoptik in Munich, Griffith University in Brisbane, Communications Research Laboratory in Tokyo, BT in Ipswich and a number of UK Universities through the EPSRC-funded Quantum Optics Network. It follows that the research experiences gained from the proposed work will be quickly disseminated throughout a large community, with an inevitable and beneficial cross-fertilisation of ideas.

Recent publications by Professor S. M. Barnett and Dr. C. Baxter have appeared in Physical Review Letters, Physical Review, Optics Letters, Optics Communications and Annals of Physics among others. Also, articles for the general reader have been written by the same authors for such magazines as New Scientist and Physics World. Thus, it is our intention that the results of the proposed research will appear in one or more of the above refereed journals, as well as been given directly to the beneficiaries through seminar and conference presentations (see Section F).

We have also recently established a website

<http://cnqo.phys.strath.ac.uk/~colinb/rp/index-rp.html>

devoted to the mechanical effects of radiation. This will be used to disseminate the research results quickly and in an easily accessible form.

We intend to write from time to time appropriate articles for the non-specialist.

Although it is unlikely that any patent or intellectual property rights would accrue directly from the proposed research, the University of Strathclyde, should the need arise, has appropriate personnel and administration procedures dedicated to the protection and exploitation of such rights. The University of Strathclyde has an outstanding reputation for excellence in the commercial exploitation of research. We will be able, should the opportunity arise, to draw on the expertise of the Research and Consultancy Services of the University of Strathclyde not only for the licensing of intellectual property but also for the development of research partnerships and consultancy projects.

## **E. The Programme**

### **1. Summary of the Background Physics**

Modern ideas about radiation pressure can be traced back to the work of Maxwell, who showed that the forces on material bodies could be calculated with the use of his electromagnetic theory [13]. The predicted magnitudes of the forces on a highly-reflecting mirror were verified in experiments by Nichols and Hull [14].

The forces associated with light beams propagated through transparent dielectric media have also been studied. Poynting [4] calculated the force on the surface of a lossless dielectric, but his calculation is difficult to follow and seems to predict, contra to intuition, an outwards force on the surface for both directions of propagation of the light. Outward forces have indeed been observed experimentally [15] in the passage of laser beams through a water surface, but these are explainable in terms of transverse forces associated with the Gaussian intensity cross-section of a typical laser beam [16]. It is clearly unsatisfactory that the nature of the force is uncertain even for the simplest form of light beam with a plane wavefront.

Another problematic area concerns the force on a highly-reflecting mirror suspended in a transparent liquid. A thorough experimental study [17] has encouraged the idea that the correct value for photon momentum is the so-called Minkowski value rather than that determined by Abraham. The connections between the photon momentum and the forces exerted by light beams on mirrors have not, however, been unambiguously established. Although the canonical momentum of light interacting with a collection of charges is consistent with the Minkowski value [18], it has been emphasised that electromagnetic momenta within materials come in different varieties [8]. A too-naïve identification of photon momentum should be avoided.

Developments in the field of quantum optics have renewed the interest in the mechanical effects of radiation. Some of the major areas have been sketched in Section A.

In general, the quantum attack on the problem of radiation pressure at interfaces has followed one of two complementary techniques. The first is to use a microscopic derivation of the electromagnetic field, together with appropriate boundary conditions, to produce a canonical set of photon creation and destruction operators [19]. The second technique uses Green's functions to obtain quantisation at the boundary [12]. It is our intention to use each method as and when appropriate.

Our proposed research will provide clear answers to some of the basic questions that are as yet unresolved by the many previous analyses. Thus, the work will determine the direction and magnitude of the force exerted on the surface of a dielectric by the incidence of one photon. This will allow a comparison to be made with Poynting's work [4]. The proposed calculations will determine the relationships between the Maxwell stress tensor, the momentum density expressions of Abraham, Minkowski and others, and the forces on free charges and mirrors in dielectric materials. The results should show the usefulness of describing the electromagnetic pressure in terms of quantum mechanics.

### **2. Plan of Work**

The immediate aims of the project are given below in items i to v.

#### **i. Setting up the General Theory**

The quantum theory will be set up for single-photon propagation at normal incidence through an interface bounding a semi-infinite dielectric medium, and through two surfaces bounding a dielectric slab. The formalism for the theory will come from [11], [20]. A general dielectric with arbitrary dispersion and loss will be assumed. It will be assumed that the light will have plane wavefronts; and

transverse effects will therefore not obscure the fundamental longitudinal radiation pressure. For analytical ease, a Gaussian shape will be assumed for the light pulse.

### **ii. Single Semi-Infinite Dielectric**

The force on a semi-infinite dielectric as the optical pulse enters through its vacuo interface will be determined from the Lorentz equation. An integration of the force over the whole dielectric medium will give the radiation pressure operator as a function of time. The dynamical development of the radiation pressure, as the pulses enter the material, will be obtained by calculating the time-dependence of the expectation value of the pressure operator. A check on the calculation will be provided by the known result in the case of a purely imaginary refractive index [10].

### **iii. Additional Forces**

The additional forces that occur in the presence of a free charge in the dielectric will be evaluated. There is an important limit in the case of a low-loss dielectric, where the system models the photon-drag experiments performed on semiconductors at frequencies below their absorption resonances [21]. The theoretical predictions will be compared with the experimental results. These particular experimental results have provided so far the most decisive measurements of the transfer of momentum between photons and charge carriers.

### **iv. Two Dielectric Media**

Similar calculations will be undertaken for two dielectrics in contact at a plane interface. These calculations will be performed using the results of [22]. The time-dependent radiation pressure operator will be determined for a pulse incident from a dispersive and lossless dielectric towards the interface with a dispersive but lossy dielectric. The expectation value of the pressure operator will be calculated for a short pulse so that the dynamics of the pressure on the second dielectric can be obtained. The limit of the second dielectric, in the form of a perfectly reflecting mirror, will produce results that can be compared to experiment [17].

### **v. Pulse Through a Dielectric**

The above calculations will be extended to the situation where a pulse passes through a dielectric slab. This system is considerably more complicated. There are now three characteristic length-scales: the length of the pulse; the thickness of the slab; the attenuation distance. Further, the effects of multiple reflections have to be considered. The lossless slab has been treated previously [9]; the new calculations will reveal additional features caused by the presence of absorption.

### **vi. Further Calculations**

Apart from the items i–v, the direction of the research cannot be foreseen. It is likely that the presence of absorption, as indicated above in v will produce a totally new set of questions. These will be explored by further calculations in the remaining time.

It is anticipated that the above “benchmarks” i–iv will be attained by the end of the first year of the proposed two-year research programme; the remaining items, v and vi, by the end of the second year. This is illustrated in diagrammatic form in *Case for Support*, Part 3. Provision has been made under vi to explore the immediate opportunities that are evident towards the end of the research contract.

## **F. Management and Resources**

The complexity of the research warrants the appointment of a full-time Research Assistant, who will carry out the above programme of research. It is essential that we employ Dr. C. Baxter in this capacity. He has a highly appropriate background in quantum electrodynamics and the key interest in radiation pressure problems. He is currently writing a review paper [23] on radiation pressure. This will provide a singular basis for the work envisaged in the present proposal. We therefore request funds in order to employ Dr. C. Baxter for a period of two years, beginning at salary point 13.

The progress of the work will be reviewed through weekly meetings between the RA and Professor S. M. Barnett. The management and control of the programme will rest with Professor S. M. Barnett.

We request the sum of £2431 to meet the cost of computer and printer equipment detailed in EPS (eRP). This equipment will be used to run the radiation-pressure website, mentioned in Section D, and for preparing papers, etc. for publication.

Finally, the sum of £2400 is requested to meet the costs of essential travel. This is again detailed in EPS (eRP). It is anticipated that the travel costs will include one trip each to Paris, Sussex, Los Alamos, and two trips to Essex to discuss with colleagues (see Section C), together with attendance at two major international conferences, in order to make formal presentations of the results of our proposed research. Attendance at such conferences, which, of course, enhances the research profile of the United Kingdom, is an essential pivot in the dissemination of our work and will consolidate its international awareness. To maximise the scope for dissemination, we intend to participate in a broad-based meeting covering all aspects of quantum electronics and laser science, as well as one dedicated to the fundamental theory of quantum optics. We anticipate that the UK Quantum Electronics Conference, 2001, and the CLEO/Quantum Electronics and Laser Science Conference, Glasgow, 2002 will form the appropriate and respective choices.

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