

Electromagnetic Tensors and the Photon Drag Effect

Stephen M. Barnett¹ C. Baxter¹ Rodney Loudon²

¹Department of Physics,
University of Strathclyde,
GLASGOW, G4 0NG, Scotland.

²Department of Electronic Systems Engineering,
University of Essex,
COLCHESTER, CO4 3SQ, England.

Physics 2005: A Century after Einstein,
University of Warwick, England.
10-14 April 2005

Introduction

A number of electromagnetic tensors were introduced at the beginning of the last Century.

Examples include the Abraham, Minkowski and Einstein-Laub tensors.

There have been various conflicting claims about the validity of these tensors, and in particular their relations to various forms of electromagnetic momentum density.

Our Argument

- ▶ We argue that there is no conflict between the various formulations of the electromagnetic stress tensor.
- ▶ There is no unique tensor formulation for the momentum transfer of light to a spatially stationary, material bulk-medium.
- ▶ The terms Abraham and Minkowski momenta should be regarded as semantic legacy-labels. Both momenta can be observed in principle.

Einstein-Laub Tensor — A Non-Canonical Tensor

A. Einstein and J. Laub, *Ann. der Phys.* **26**, 541 (1908)

- ▶ Formulated by considering reaction of a material dipole to electromagnetic fields.
- ▶ Enables determinations of electrostriction and magnetostriction effects — original motivation. [★]
- ▶ One of a number of electromagnetic tensors whose formulation is non-canonical, *i.e.* from direct physical considerations and equations of motion.
- ▶ Other non-canonical electromagnetic tensors include the Peierls tensor. [†]

★ I. Brevik, *Phys. Rep.* **52**, 133 (1979)

† R. Peierls, *Proc. Soc. Lond.* **355**, 141 (1977)

Canonical Determination of Electromagnetic Tensors

L. D. Landau and E. M. Lifshitz, "The Classical Theory of Fields", Pergamon Press (Oxford) 1975

- ▶ Variation of Lagrangian with respect to spacetime metric – depends on form of Lagrangian – bulk media only.
- ▶ Two ways of constructing Lagrangian for EM fields in material medium.
 1. Incorporate ϵ, μ in an effective metric.
 - ▶ applicable to spatially non-stationary media
 - ▶ ϵ, μ must be real scalars
 - ▶ leads initially to Minkowski tensor
 - ▶ "correcting" effective metric to actual (Minkowski) metric leads to Abraham tensor ‡
 2. Incorporate polarisation currents in a minimal-coupling term.
 - ▶ medium is spatially stationary
 - ▶ leads to Symmetric tensor

‡ S. Antoci and L. Mihich, *Nuovo Cim. B* **112**, 991 (1997)

Material Medium — Optical Experiments

We consider the use of tensors relevant to optical experiments.

- ▶ Medium is usually non-magnetic.
- ▶ Medium is spatially stationary. Abraham and Minkowski tensor formulations are therefore equivalent. \oplus

\oplus Here, the effective and Minkowski metrics differ only in their scalar component. This effectively means that the Abraham and Minkowski tensor formulations are simply related by a gauge transformation.

Abraham Tensor — Spatially Stationary Medium

M. Abraham, *C. R. Circ. Mat. Palermo* **28**, 1 (1909); *ibid.* **30** 33 (1910)

- ▶ Spatial Components

$$T_A^{ij} = \frac{1}{2} \{ \delta^{ij} \mathbf{E} \cdot \mathbf{D} - E^i D^j - E^j D^i \} + \frac{1}{2} \{ \delta^{ij} \mathbf{H} \cdot \mathbf{B} - H^i B^j - H^j B^i \}$$

- ▶ Momentum Density

$$\mathbf{g}_A = \{ \mathbf{E} \times \mathbf{H} \}^i / c^2$$

- ▶ Conservation of Linear Momentum

$$\frac{\partial g_A^j}{\partial t} + \sum_{i=1}^3 \frac{\partial T_A^{ij}}{\partial x^i} = -f_A^j$$

- ▶ Abraham force density

$$\mathbf{f}_A = \epsilon_0 \chi \frac{\partial}{\partial t} (\mathbf{E} \times \mathbf{B})$$

Minkowski Tensor — Spatially Stationary Medium

H. Minkowski, *Nachr. Ges. Wiss. Göttinger* 55 (1908); *Math. Ann.* **68** 472 (1910)

- ▶ Spatial Components

$$T_M^{ij} = \frac{\delta^{ij}}{2} \mathbf{E} \cdot \mathbf{D} - E^i D^j + \frac{\delta^{ij}}{2} \mathbf{H} \cdot \mathbf{B} - H^i B^j$$

- ▶ Momentum Density

$$g_M^i = \{\mathbf{D} \times \mathbf{B}\}^i$$

- ▶ Conservation of Linear Momentum

$$\frac{\partial g_M^j}{\partial t} + \sum_{i=1}^3 \frac{\partial T_M^{ij}}{\partial x^i} = 0$$

Symmetric (Maxwell) Tensor

J. D. Jackson, "Classical Electrodynamics" Wiley (New York) 1999

- ▶ Spatial Components

$$T_S^{ij} = \epsilon_0 \left\{ \frac{\delta^{ij}}{2} \mathbf{E}^2 - E^i E^j \right\} + \frac{1}{\mu_0} \left\{ \frac{\delta^{ij}}{2} \mathbf{B}^2 - B^i B^j \right\}$$

- ▶ Momentum Density

$$\mathbf{g}_S = \epsilon_0 \{ \mathbf{E} \times \mathbf{B} \}^v$$

- ▶ Conservation of Linear Momentum

$$\frac{\partial g_S^j}{\partial t} + \sum_{i=1}^3 \frac{\partial T_S^{ij}}{\partial x^i} = -f_L^j$$

- ▶ Lorentz force density

$$\mathbf{f}_L = (\mathbf{P} \cdot \nabla) \mathbf{E} + \left\{ \frac{\partial \mathbf{P}}{\partial t} \right\} \times \mathbf{B}$$

Einstein-Laub Tensor

A. Einstein and J. Laub, *Ann. der Phys.* **26**, 541 (1908)

- ▶ Spatial Components

$$T_{EL}^{ij} = \frac{\delta^{ij}}{2} \epsilon_0 \mathbf{E}^2 - E^i D^j + \frac{\delta^{ij}}{2} \mu_0 \mathbf{H}^2 - H^i B^j,$$

- ▶ Momentum Density

$$g_{EL}^i = \{\mathbf{E} \times \mathbf{H}\}^i / c^2 = \text{Abraham momentum } (g_A^i)$$

- ▶ Conservation of Linear Momentum

$$\frac{\partial g_A^j}{\partial t} + \sum_{i=1}^3 \frac{\partial T_{EL}^{ij}}{\partial x^i} = -f_L^j$$

Tensors Provide Equivalent Information

Spatially Stationary Bulk-Medium

- ▶ Momentum Conservation provides equivalent tensor equations.
- ▶ No one tensor is necessarily correct and others wrong.
- ▶ No one momentum is necessarily to be preferred.
- ▶ All components of a tensor need to be used.
- ▶ Inconsistencies will arise if momentum densities are isolated and considered separately (Abraham-Minkowski controversy).

Explicit Illustration of Equivalence of Tensors by means of the Photon Drag Effect

The photon drag effect:-

- ▶ One of the simplest manifestations of photon pressure.
- ▶ Generates an electric current in a semi-conductor by transferring momentum from an incident light beam to the charge carriers.
- ▶ A phenomenon of the bulk semiconductor.

Momentum transfer has been observed to be the Minkowski value. ~

~ A. F. Gibson *et al*, *Proc. Roy. Soc. Lond.* **370**, 303 (1980)

Photon Drag — Simple Analysis

- ▶ Light field varies with only one spatial co-ordinate z .
- ▶ Maxwell's equations simplify to:

$$\frac{\partial E}{\partial z} + \frac{\partial B}{\partial t} = 0, \quad \frac{\partial H}{\partial z} + \frac{\partial D}{\partial t} = 0.$$

- ▶ Solution is $B = \eta E/c$ with loss and dispersion ignored.
- ▶ Momentum conservation equations involving Abraham, Minkowski, Symmetric and Einstein-Laub tensors reduce to single equation

$$\frac{\partial}{\partial t} \left[\frac{\eta U}{c} \right] + \frac{\partial U}{\partial z} = 0.$$

η = refractive index

U = energy density

Photon Drag — Conclusions

- ▶ Light transfers momentum of the Minkowski value $\eta U/c$ in the photon drag effect.
- ▶ The Minkowski tensor formulation provides the **simplest** – but not the **only** – division of momentum within the bulk semi-conductor.
- ▶ Provided momentum densities are not used in isolation, other tensor formulations will produce exactly same result.

Photon Drag — Further Analysis

Tensor formulations in light pressure calculations are limited to bulk material media. Dispersion and loss cannot be easily incorporated.

Detailed analysis[§] of the photon drag effect, involving the Lorentz force operator and a quantised light pulse, shows for a lossy and dispersive semi-conductor:-

- ▶ Light transfers momentum of the Minkowski value to the charge carriers.
- ▶ Temporal separation occurs between the surface and bulk contributions to the forces when the pulse is much shorter than the attenuation length. In this case, the total momentum transfer to the charges **and** semi-conductor host material is of the Abraham value.

§ Rodney Loudon, Stephen M. Barnett, C. Baxter, Preprint, Submitted to *Phys. Rev. A* (2005)

Final Conclusions

- ▶ There is no conflict between the various formulations of the electromagnetic stress tensor.
- ▶ There is no unique tensor formulation for the momentum transfer of light to a spatially stationary, material bulk-medium.
- ▶ The terms Abraham and Minkowski momenta should be regarded as semantic legacy-labels. Both momenta can be observed.