

States insensitive to the Unruh effect in multilevel detectors

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We give a general treatment of the spontaneous excitation rates and the non-relativistic Lamb shift of constantly accelerated multilevel atoms as a model for multilevel detectors. Using a covariant formulation of the dipole coupling between the atom and the electromagnetic field we show that new Raman-like transitions can be induced by the acceleration. Under certain conditions these transitions can lead to stable ground and excited states that are not affected by the noninertial motion. The magnitude of the Unruh effect is not altered by multilevel effects. Both the spontaneous excitation rates and the Lamb shift are not within the range of measurability. [S0556-2821(98)04302-1]

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I. INTRODUCTION

The Unruh effect [1] is the prediction that a linearly accelerated two-level particle detector that is sensitive to detect real massless particles becomes excited when moving through the Minkowski vacuum. It behaves as if it were in a thermal bath of particles with Unruh temperature $T_U = \hbar a / (2\pi k_B c)$, where a is the acceleration of the detector. Although this temperature is in general extremely small the Unruh effect gained much interest in theoretical physics. For a detailed discussion see, for example, Ref. [2]. After Unruh and Wald [3] reported acausal correlations and lack of energy conservation the emission of radiation by the detector has been questioned [4]. The discussion was resolved by the inclusion of the measurement process [5]. For a two-level atom coupled to a scalar field the spontaneous excitation of a uniformly accelerated atom in its ground state was interpreted in terms of vacuum fluctuations and radiation reaction by Audretsch and Müller [6]. The associated Lamb shift was worked out in Ref. [7], and arbitrary stationary trajectories including rotation were discussed in Ref. [8]. This quantum optical approach is similar to the one used in the present paper. The experimental realizability was discussed by Bell and Leinaas [9] who considered the spin of an electron in a storage ring instead of a detector model. Since under these conditions the acceleration can be extremely large the Unruh temperature should reach values of about 1000 K, but unfortunately the fluctuations of the electron's trajectory in the storage ring cover the effect. With regard to the physical interpretation the rotating particle detector as well as the rotating quantum vacuum pose deep conceptual problems [10,11].

It remains therefore a challenge to propose quantum systems for which the Unruh effect or a similar effect becomes measurable. In this connection it has been conjectured that multilevel atoms with their variety of complex transitions

may show acceleration-induced effects that are considerably bigger than the usual Unruh effect. In this paper we will give an answer to this question. To discuss realistic situations as far as the coupling is concerned, we base our calculations on the relativistic generalization of the electromagnetic dipole coupling. This will give as a by-product an improved expression for the Lamb shift of accelerated atoms.

The paper is organized as follows. In Sec. II we introduce the dipole coupling to the electromagnetic field for arbitrary atom trajectories. In Sec. III we derive the Master equation for the density matrix of moving atoms. For later use we review the Unruh effect for the electromagnetic coupling in Sec. IV and derive the corresponding Lamb shift. Finally we turn to acceleration-induced multilevel effects in Sec. V.

II. THE COUPLING

We consider atoms forced to move on a given classical trajectory $z^\mu(t)$ with proper time t . The interaction with the electromagnetic field is described in the dipole approximation and consists of a relativistic generalization of the dipole coupling [12,2],

$$H_{\text{int}} = -c \dot{z}^\mu(t) d^\nu F_{\mu\nu}(z^\nu(t)). \quad (1)$$

$\mu, \nu, \dots = 0, \dots, 3$. We use the summation convention throughout the paper if not otherwise stated. A dot denotes the derivative with respect to t . $F_{\mu\nu}$ is the electromagnetic field strength operator, and d^ν is the dipole operator, which describes the internal induced dipole moments of the atom.

If one replaces $F_{\mu\nu}$ by the electric and magnetic field one can see that, in the laboratory frame, the coupling (1) does not only include the usual dipole coupling to the electric field but also the Röntgen term [13], which represents a coupling between the magnetic field, the atomic center-of-mass momentum, and its internal dipole moment. Its importance for the relativistic invariance of the spontaneous decay rate has been demonstrated by Wilkens [14]. A microphysical derivation of the dipole coupling and the Röntgen term including the effect of a weak gravitational field was done in Ref. [15].

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To describe the 4-velocity and the local rest frame of the atom, we introduce along the trajectory a set of four orthonormalized vectors $e_{\underline{\alpha}}^{\mu}(t)$, $\underline{\alpha}=0,1,2,3$, with $e_{\alpha\mu}e_{\beta}^{\mu} = \text{diag}(-1,1,1,1)$ (see, e.g., Ref. [16]). Underlined indices represent tetrad numbers. $e_0^{\mu}(t)$ is the tangent $\dot{z}^{\mu}(t)$ to the worldline. The three spacelike vectors $e_{\underline{a}}^{\mu}(t)$, $\underline{a}=1,2,3$, are Fermi propagated along the trajectory. With respect to the tetrad the dipole operator is given by $d_{\underline{\alpha}} = e_{\underline{\alpha}} \cdot d \equiv e_{\underline{\alpha}}^{\mu} d_{\mu}$. In the Schrödinger picture the internal dipole moments are constant. We go to the interaction picture. The Schrödinger equation then reads

$$i\hbar \frac{d\psi}{dt} = -cd^{\underline{a}}(t)F_{\underline{0a}}(z^{\mu}(t))\psi. \quad (2)$$

Because $F_{\underline{00}}$ always vanishes the expression $d^{\underline{a}}(t)F_{\underline{0a}}$ could be reduced to $d^{\underline{a}}(t)F_{\underline{0a}}$. Equation (2) is manifestly covariant as well as gauge invariant.

The components $d^{\underline{a}}(t)$ of the dipole moment operator can be written as

$$d^{\underline{a}}(t) = \sum_{i,j=1}^N d_{ij}^{\underline{a}} |i\rangle\langle j| \exp[i\omega_{ij}t], \quad (3)$$

where $|i\rangle$, $i,j=1, \dots, N$, are the time-independent internal energy eigenstates of an atom with N internal levels with energy E_i . We assume that the effect of the acceleration on the internal states and the energy levels has already been included. The transition frequencies are given by $\omega_{ij} := (E_i - E_j)/\hbar$. Since we are working in the interaction picture each coherence operator is oscillating with frequency ω_{ij} , and since the Schrödinger equation (2) is written with respect to the atom's proper time these oscillations still have the form $\exp[i\omega_{ij}t]$.

The vector components $d_{ij}^{\underline{a}}$ are constant because they refer to the frame $e_{\underline{\alpha}}^{\mu}$ attached to the atom. They are given by

$$\vec{d}_{ij} = e \langle i | \vec{x} | j \rangle \quad (4)$$

for a one-electron atom, where e is the charge of the electron and \vec{x} its position operator relative to the center-of-mass position. Any spatial vector \vec{v} is defined by having the components $v^{\underline{b}}$, $\underline{b}=1,2,3$, with respect to the tetrad. Because $d^{\underline{a}}(t)$ has to be Hermitean we have $d_{ij}^{\underline{a}} = d_{ji}^{\underline{a}*}$ and because of parity conservation $d_{ii}^{\underline{a}} = 0$ holds.

To quantize the Maxwell field we introduce the electromagnetic 4-potential A_{μ} with $F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}$. The laboratory frame is an inertial reference frame. With regard to the concept of a photon and accordingly to the concept of the *Minkowski vacuum*, we refer to this frame and decompose the 4-potential in the Lorentz gauge as follows:

$$A^{\mu}(x^{\nu}) = \sqrt{\frac{\hbar}{2(2\pi)^3 \varepsilon_0 c}} \int \frac{d^3k}{\sqrt{k}} \{ \exp(ikx) \varepsilon_{\underline{\alpha}}^{\mu}(\mathbf{k}) a^{\underline{\alpha}}(\mathbf{k}) + \text{H.c.} \}, \quad (5)$$

with $k^{\mu} := (k \equiv |\mathbf{k}|, \mathbf{k})$ and \mathbf{k} being the wave vector in the laboratory frame. $a^{\underline{\alpha}}(\mathbf{k})$ are the usual annihilation operators fulfilling $[a^{\underline{\alpha}}(\mathbf{k}), a^{\underline{\beta}}(\mathbf{k}')] = \delta(\mathbf{k} - \mathbf{k}') \eta^{\underline{\alpha}\underline{\beta}}$. The vectors $\varepsilon_{\underline{\alpha}}^{\mu}(\mathbf{k})$ are the (real) normalized polarization vectors of the electromagnetic field and indices with carets are polarization indices, $\hat{\alpha}=0,1,2,3$. $\varepsilon_0^{\mu}(\mathbf{k})$ corresponds to the scalar photon, $\varepsilon_3^{\mu}(\mathbf{k})$ to the longitudinal photon, and $\varepsilon_1^{\mu}(\mathbf{k})$ as well as $\varepsilon_2^{\mu}(\mathbf{k})$ to the two transverse degrees of freedom related to the physical photons (compare, e.g., Ref. [17]). Note that for each \mathbf{k} these four vectors do provide an orthonormal tetrad at rest adjusted to the photon ray with wave vector \mathbf{k} . When working out the influence of the acceleration of the atoms we will not only have to take into account their noninertial 4-trajectory but also the orientation of the rest-frame tetrad $e_{\underline{\alpha}}^{\mu}(t)$ relative to the laboratory tetrad $\varepsilon_{\underline{\alpha}}^{\mu}(\mathbf{k})$. On this basis we will now formulate the quantum optics of multilevel atoms moving with uniform acceleration through the Minkowski vacuum $|0\rangle$ with $a^{\underline{\alpha}}(\mathbf{k})|0\rangle = 0$.

III. MASTER EQUATION FOR MOVING ATOMS

In this paper we will use a master equation to calculate the decay rates and the non-relativistic part of the Lamb shift for a given energy level. The master equation technique is described in many textbooks on quantum optics (see, e.g., Ref. [18]). In short its central assumption is that the density matrix ρ_{tot} for the atom-field system can approximately be decomposed into an atomic density matrix $\rho(t)$ and a field density matrix (in our case the Minkowski vacuum), which remains constant, $\rho_{\text{tot}}(t) = \rho(t) \otimes |0\rangle\langle 0|$. In second order of perturbation theory one can derive the following master equation for the atomic density matrix,

$$\frac{d\rho(t)}{dt} = \frac{-1}{\hbar^2} \int_0^t dt' \{ [d^{\underline{a}}(t), d^{\underline{b}}(t')] \rho(t)] G_{\underline{ab}}(t, t') - [d^{\underline{a}}(t), \rho(t) d^{\underline{b}}(t')] G_{\underline{ab}}^*(t, t') \}, \quad (6)$$

with the correlation function

$$G_{\underline{ab}}(t, t') := c^2 \langle 0 | F_{\underline{0a}}(z^{\nu}(t)) F_{\underline{0b}}(z^{\nu}(t')) | 0 \rangle. \quad (7)$$

Using Eqs. (5) the function $G_{\underline{ab}}(t, t')$ can be written as

$$G_{\underline{ab}}(t, t') = \frac{\hbar c}{2(2\pi)^3 \varepsilon_0} \int \frac{d^3k}{k} e^{ik[z(t) - z(t')]} \{ k_0(t) k_0(t') [e_{\underline{a}}(t) \cdot e_{\underline{b}}(t')] - k_0(t) k_{\underline{b}}(t') [e_{\underline{a}}(t) \cdot e_0(t')] - k_{\underline{a}}(t) k_0(t') [e_0(t) \cdot e_{\underline{b}}(t')] + k_{\underline{a}}(t) k_{\underline{b}}(t') [e_0(t) \cdot e_0(t')] \}. \quad (8)$$

Its precise form depends on the atomic trajectory $z^\mu(t)$. The trajectory manifests itself also in the projections on the co-moving (Fermi propagated) tetrad. To solve the master equation it is convenient to expand ρ as

$$\rho(t) = \sum_{i,j=1}^N |i\rangle\langle j| \rho_{ij}(t) \quad (9)$$

and to insert this in Eq. (6). We will describe the stationary situation of inertially moving atoms or atoms in constant acceleration. The time dependence in Eq. (8) reduces therefore to $t'' = t - t'$ and we can make the commonly used approximation to extend the resulting integral from 0 to ∞ instead from 0 to t . This is well justified if $G_{\underline{ab}}(t)$ falls off rapidly. Defining

$$K_{ij,kl}(\omega) := d_{ki}^a d_{lj}^b \int_0^\infty dt'' e^{-i\omega t''} G_{\underline{ab}}(t'') \quad (10)$$

one finds the master equation

$$\begin{aligned} \frac{d\rho_{ij}}{dt} = & \frac{-1}{\hbar^2} \sum_{k,l=1}^N \{ \rho_{lj} e^{i\omega_{lj} t} K_{ik,kl}(\omega_{kl}) + \rho_{ik} e^{i\omega_{kj} t} K_{jl,lk}^*(-\omega_{kl}) \\ & - \rho_{kl} e^{it(\omega_{ik} + \omega_{lj})} [K_{lj,ik}(\omega_{ik}) + K_{ki,jl}^*(-\omega_{lj})] \}. \end{aligned} \quad (11)$$

We will see below that the two-point function (7) is isotropic, $G_{\underline{ab}}(t) = \delta_{\underline{ab}} G(t)$. Accordingly we can reduce Eq. (10) to

$$K_{ij,kl}(\omega) = (\vec{d}_{ij} \cdot \vec{d}_{kl}) K(\omega). \quad (12)$$

The specific trajectory will manifest itself in the form of the function $K(\omega)$.

IV. UNRUH EFFECT AND LAMB SHIFT FOR TWO-LEVEL ATOMS

We study two-level atoms first to derive the modification of the Lamb shift caused by the acceleration for the interaction of the atom with the inertial electromagnetic instead of the scalar vacuum. In doing so we will as well provide the expressions on which the discussion of multilevel atoms can be based. We briefly review the inertial situation as well because we have to refer to it for the renormalization.

For a two-level atom with an excited state $|e\rangle$ and a ground state $|g\rangle$ two components in the dipole operator, namely, d_{eg}^a and $d_{ge}^a = d_{eg}^{a*}$, do not vanish. The master equation (11) then becomes

$$\frac{d\rho_{gg}}{dt} = \frac{2|\vec{d}_{eg}|^2}{\hbar^2} \{ \rho_{ee} \text{Re}K(-\omega_{eg}) - \rho_{gg} \text{Re}K(\omega_{eg}) \}, \quad (13)$$

$$\begin{aligned} \frac{d\rho_{eg}}{dt} = & -\rho_{eg} \frac{|\vec{d}_{eg}|^2}{\hbar^2} [K(-\omega_{eg}) + K^*(\omega_{eg})] \\ & + \rho_{ge} \frac{\vec{d}_{eg}^2}{\hbar^2} e^{2i\omega_{eg} t} [K(\omega_{eg}) + K^*(-\omega_{eg})] \end{aligned} \quad (14)$$

together with the normalization condition $\rho_{ee} + \rho_{gg} = 1$. The term proportional to ρ_{ee} in Eq. (13) describes the spontaneous emission of a photon and the corresponding deexcitation of the atom and will be present also for inertially moving atoms. The term proportional to ρ_{gg} describes spontaneous excitation. It vanishes for inertial atoms and displays the Unruh effect for accelerated atoms. Equation (14) is mainly of interest for the imaginary part of the coefficient connected with ρ_{eg} . In the Schrödinger picture the unperturbed coherence ρ_{eg} oscillates with the transition frequency ω_{eg} and hence gives us information about the energy difference between the two states. An imaginary part in the interaction picture indicates that the oscillation frequency and therefore the energy difference has changed. It describes the level shift caused by the interaction with the atom and can be interpreted as the non-relativistic Lamb shift.

Inertial atoms: The world line of the inertially moving atom is given by $z^\mu(t) = \bar{z}^\mu + ct e_0^\mu$. The tetrad vectors e_α^μ are constant for this trajectory. Changing in Eq. (8) the integration variables to $p_c := k_\epsilon$ one can exploit the invariance of the integration measure under Lorentz boosts [19], i.e., $d^3k/k = d^3p/p$ with $p := |\vec{p}| = -k_0$, to simplify Eq. (7) to $G_{\underline{ab}}^{\text{inertial}} = \delta_{\underline{ab}} G^{\text{inertial}}$ with

$$G^{\text{inertial}}(t-t') = \frac{-\hbar}{6\pi^2 \epsilon_0 c^3} \frac{d^3}{dt^3} \left\{ i\pi \delta(t-t') + \frac{\mathcal{P}}{t-t'} \right\}. \quad (15)$$

Here \mathcal{P} denotes the principal value (in the sense of distributions) and we have used

$$\int_0^\infty dp e^{-ipx} = \pi \delta(x) - i \frac{\mathcal{P}}{x}. \quad (16)$$

Inserting this into Eq. (12) and using $G_{\underline{ab}}^*(t) = G_{\underline{ab}}(-t)$ (for all cases considered in this paper) we arrive at

$$\begin{aligned} 2 \text{Re}K^{\text{inertial}}(\omega) &= \int_{-\infty}^\infty dt e^{-i\omega t} G^{\text{inertial}}(t) \\ &= \theta(-\omega) \frac{\hbar |\omega|^3}{3\pi \epsilon_0 c^3}, \quad i, j = 1, 2, 3. \end{aligned} \quad (17)$$

In the derivation we used the residue theorem and the fact that for functions $f(z)$ with a first-order pole at $z=0$ the expression $\int_{-\infty}^\infty dx \mathcal{P}f(x)$ can be replaced by $(1/2) \lim_{\epsilon \rightarrow 0} \int_{-\infty}^\infty dx \{ f(x+i\epsilon) + f(x-i\epsilon) \}$. According to Eq. (13) the spontaneous emission rate of an excited atom is given by the first term in Eq. (13), $2|\vec{d}_{eg}|^2 \text{Re}K(-\omega_{eg})/\hbar^2$, which results in

$$\Gamma_{\text{inertial}}^{e \rightarrow g} = \frac{\omega_{eg}^3 |\vec{d}_{eg}|^2}{3\pi \hbar \epsilon_0 c^3}. \quad (18)$$

This coincides with the classic result of Wigner and Weisskopf (see, e.g., [20]). On the other hand, we can infer from the second term in Eq. (13) that inertial atoms in vacuum do not suffer from spontaneous excitation in the ground state

because the corresponding transition rate $\Gamma_{\text{inertial}}^{g \rightarrow e}$ is proportional to $\text{Re}K^{\text{inertial}}(\omega_{eg})$ and thus vanishes because of $\theta(-\omega)$ in Eq. (17).

We have to remark that the imaginary part of $K^{\text{inertial}}(\omega)$,

$$\text{Im}K^{\text{inertial}}(\omega) = \frac{\hbar \omega^3}{6\pi^2 \varepsilon_0 c^3} \int_0^\infty dt \frac{\mathcal{P}}{t} \cos(\omega t), \quad (19)$$

is divergent. To renormalize it we refer to the previously mentioned fact that the imaginary part corresponds to an energy level shift $\Delta E_{\text{inertial}}$. We therefore set as renormalization $\text{Im}K^{\text{inertial}}(\omega) = \hbar \Delta E_{\text{inertial}} / (2|\vec{d}_{eg}|^2)$. The inertial energy shift cannot be calculated within the two-level atom approximation and must be adopted from relativistic theories (see, e.g., Ref. [20]).

Accelerated atoms: We now turn to uniformly accelerated atoms moving through the Minkowski vacuum on the well-known Rindler trajectory

$$z^\mu(t) = \frac{c^2}{a} (\sinh[at/c], 0, 0, \cosh[at/c]), \quad (20)$$

where $a > 0$ is the constant acceleration. The tetrad vectors e_1^μ and e_2^μ carried with the atom do always point in the 1- and 2-direction of the laboratory frame. The remaining vectors are given by $e_0^\mu = (\cosh[at/c], 0, 0, \sinh[at/c])$ and $e_3^\mu = (\sinh[at/c], 0, 0, \cosh[at/c])$. The two-point function can be calculated similarly to the inertial case by changing the integration variables in Eq. (8) to $p_i = k_i[(t+t')/2]$. The two-point functions have been derived in section 9.3 of Ref. [2] in the context of a detector model with the result

$$c^2 \langle 0 | F_{0\bar{a}}(t) F_{0\bar{b}}(t') | 0 \rangle = \delta_{\bar{a}\bar{b}} G(t-t'), \quad (21)$$

where within our model the distribution G is defined by

$$G(t-t') := \frac{-\hbar a^4}{96\pi^2 \varepsilon_0 c^7} \frac{d^3}{du^3} \left\{ i\pi \delta(u) + \frac{\mathcal{P}}{u} \right\}_{u=\sinh(\Delta)}, \quad (22)$$

with $\Delta := (t-t')a/(2c)$. The derivative of the δ function and the principal value is defined in the sense of distributions: Let $D(x)$ be a distribution, $g(x)$ a test function, and $f(x)$ some function that is monotonic in the interval of integration. Then the derivative is defined by

$$\int dx g(x) \frac{d^n}{du^n} D(u) \Big|_{u=f(x)} = (-1)^n \int dx D(f(x)) \left(\frac{d}{dx} \frac{1}{df/dx} \right)^n g(x). \quad (23)$$

Takagi [2] also derived the power spectrum $2 \text{Re}K(\omega)$ for the detector model. Transformed into our notation it reads

$$2 \text{Re}K(\omega) = \frac{\hbar |\omega|^3}{3\pi \varepsilon_0 c^3} \left[1 + \frac{a^2}{c^2 \omega^2} \right] \left\{ \theta(-\omega) + \frac{1}{e^{2\pi|\omega|c/a} - 1} \right\}. \quad (24)$$

The spontaneous transition rates can be deduced by inserting this into Eq. (13) and are given by

$$\Gamma_{\text{local}}^{e \rightarrow g} = \Gamma_{\text{inertial}} \left[1 + \frac{a^2}{c^2 \omega_{eg}^2} \right] \left\{ 1 + \frac{1}{e^{2\pi\omega_{eg}c/a} - 1} \right\}, \quad (25)$$

$$\Gamma_{\text{local}}^{g \rightarrow e} = \Gamma_{\text{inertial}} \left[1 + \frac{a^2}{c^2 \omega_{eg}^2} \right] \frac{1}{e^{2\pi\omega_{eg}c/a} - 1}. \quad (26)$$

The ‘‘1’’ in the curly brackets of Eq. (25) is just the inertial decay rate while the second term in these brackets represents the thermal factor used to define the Unruh temperature T_U by $(\hbar|\omega|)/(k_B T_U) = 2\pi|\omega|c/a$. The additional term in square brackets is a nonthermal correction. It is caused by the coupling to the electromagnetic field strength operator instead of a structureless scalar field. The possibility of a spontaneous excitation ($\Gamma_{\text{local}}^{g \rightarrow e} > 0$) in the ground state corresponds to that phenomenon that is usually referred to as the *Unruh effect*. The nonthermal corrections for the electromagnetic dipole coupling are not in conflict with the proof that the Unruh effect is of thermal nature for all quantum fields [28] because this was shown for the equilibrium partition only. The precise statement of the proof is that, for a two-level atom, for instance, the equilibrium partition obeys $\rho_{ee}/\rho_{gg} = \exp[-\hbar\omega_{eg}/(k_B T_U)]$. It is easy to see that this relation is fulfilled by the stationary solution of Eq. (13).

Another physically interesting effect is the radiative level shift of the accelerated atom for the coupling to the Maxwell field. For a two-level atom coupled to a scalar field this has been done previously in Ref. [21]. We mentioned in Sec. III that the shift can be identified with the imaginary part of the ρ_{eg} coefficient (in the case of a two-level atom) in Eq. (14), namely,

$$\begin{aligned} \Delta E &= \frac{-1}{\hbar} \text{Im} \{ K_{eg,ge}(-\omega_{eg}) + K_{ge,eg}^*(\omega_{eg}) \} \\ &= \frac{-|\vec{d}_{eg}|^2}{\hbar} \{ \text{Im}K(-\omega_{eg}) - \text{Im}K(\omega_{eg}) \}. \end{aligned} \quad (27)$$

For the derivation of $\text{Im}K(\omega)$ we write it as $[K(\omega) - K^*(\omega)]/(2i)$ and make use of Eqs. (22) and (23). This procedure results in

$$\begin{aligned} \text{Im} K(\omega) = & \frac{\hbar a \omega^3}{12\pi^2 \varepsilon_0 c^4} \int_0^\infty dt \cos(\omega t) \frac{\mathcal{P}}{\sinh(\Delta) \cosh^3(\Delta)} \left\{ 1 + \frac{a^2}{\omega^2 c^2 \cosh^2(\Delta)} \right\} + \frac{\hbar a^4}{96\pi^2 \varepsilon_0 c^7} \int_0^\infty \frac{dt}{\cosh^4(\Delta)} \\ & \times \left\{ 3 \sin(\omega t) \left[2 \tanh^2(\Delta) - \frac{3}{\cosh^2(\Delta)} - \frac{8\omega^2 c^2}{a^2} \right] - \frac{22\omega c}{a} \tanh(\Delta) \cos(\omega t) \right\}. \end{aligned} \quad (28)$$

The first of the two integrals diverges for $t \rightarrow 0$, but the divergence is the same as for the inertially moving atom. After the use of the renormalization condition performed for $a=0$ it is therefore well defined.

To see this more explicitly we consider the case where the acceleration a is smaller than $c\omega_{eg}$. It is then a very good approximation to expand the integrand to lowest order in a . We then find

$$\text{Im}K(\omega) = \left(1 + \frac{a^2}{\omega^2 c^2} \right) \text{Im}K^{\text{inertial}}(\omega) - \frac{\hbar a^2 \omega^2}{12\pi^2 \varepsilon_0 c^5} \int_0^\infty \left\{ \frac{5\omega t}{6} \cos(\omega t) + 3 \sin(\omega t) \right\} + O(a^4). \quad (29)$$

Performing the integrations and applying the inertial renormalization condition we eventually arrive at

$$\text{Im}K(\omega_{eg}) = -\frac{13}{72\pi^2} \frac{\hbar \omega_{eg} a^2}{\varepsilon_0 c^5} + \frac{\hbar \Delta E_{\text{inertial}}}{2|\vec{d}_{eg}|^2} \left(1 + \frac{a^2}{\omega_{eg}^2 c^2} \right) + O(a^4). \quad (30)$$

The corresponding *Lamb shift* of the atoms due to the acceleration is given by

$$\Delta E = -\frac{13}{36\pi^2} \frac{a^2 \omega_{eg} e^2 |\langle e|\vec{x}|g\rangle|^2}{\varepsilon_0 c^5} + \Delta E_{\text{inertial}} \left(1 + \frac{a^2}{\omega_{eg}^2 c^2} \right) + O(a^4). \quad (31)$$

Besides the term containing the inertial Lamb shift, which is absent for a scalar field, our result coincides structurally with those of Ref. [21] for the scalar coupling. It is interesting to observe that the actual dependence of the noninertial Lamb shift on the acceleration depends on the renormalization scheme used for the inertial case [22].

Although this result is interesting for theoretical reasons the noninertial Lamb shift is far beyond the scope of present experiments: Taking for $\hbar \omega_{eg}$ a value of 10 eV and for the matrix element $|\langle e|\vec{x}|g\rangle|$ the Bohr radius of 5×10^{-11} m the first contribution to the Lamb shift of Eq. (31) in the Earth's gravitational field ($a=9.81$ m/s²) is about 10^{-54} eV. If we insert for $\Delta E_{\text{inertial}}$ the measured Lamb shift of 4.4×10^{-6} eV for the hydrogen atom the second contribution is about 2×10^{-53} eV [22].

V. ACCELERATION-INDUCED MULTILEVEL EFFECTS

We want to address three questions: As compared with the two-level atom, does a multi-level atom show new effects caused by the acceleration? What is the order of magnitude of these effects? Are there states that show no spontaneous excitation at all? To discuss this we study a three-level atom because the answers can already be read off there.

Three-level atom in Λ configuration: The Λ system consists of two ground states $|+\rangle$ and $|-\rangle$ with energies E_+ and E_- and an excited state $|e\rangle$ with energy E_e . The ground states are not directly coupled so that only $d_{+e}^a = d_{e+}^{a*}$ and $d_{-e}^a = d_{e-}^{a*}$ are nonvanishing. The Λ system is of interest because of the possibility of Raman transitions (see, e.g., Ref. [23]) between the two ground states via the excited state under the influence of an electromagnetic radiation field. These transitions can be very effective even if the driving mechanism, for instance, a laser, is far off-resonant.

Turning to accelerated atoms in the Minkowski vacuum, we display the master equation for ρ_{++} and ρ_{+-} ,

$$\begin{aligned} \frac{d\rho_{++}}{dt} = & \frac{1}{\hbar^2} \{ -2\rho_{++} |\vec{d}_{+e}|^2 \text{Re}K(\omega_{e+}) + 2\rho_{ee} |\vec{d}_{+e}|^2 \text{Re}K(-\omega_{e+}) - \rho_{+-} e^{i\omega_{+-}t} (\vec{d}_{+e} \cdot \vec{d}_{e-}) K(\omega_{e-}) \\ & - \rho_{+-} e^{-i\omega_{+-}t} (\vec{d}_{+e} \cdot \vec{d}_{e-})^* K^*(\omega_{e-}) \} \end{aligned} \quad (32)$$

$$\begin{aligned} \frac{d\rho_{+-}}{dt} = & \frac{-1}{\hbar^2} \{ \rho_{+-} [|\vec{d}_{+e}|^2 \text{Re}K(\omega_{e+}) + |\vec{d}_{-e}|^2 \text{Re}K^*(\omega_{e-})] + e^{-i\omega_{+-}t} (\vec{d}_{+e} \cdot \vec{d}_{e-}) \times \{ \rho_{--} K(\omega_{e-}) \\ & + \rho_{++} K^*(\omega_{e+}) - \rho_{ee} [K(-\omega_{e+}) + K^*(-\omega_{e-})] \} \}. \end{aligned} \quad (33)$$

The corresponding equation for ρ_{--} is gained if the $+$ and $-$ indices in Eq. (32) are interchanged. ρ_{-+} is simply the complex conjugate of ρ_{+-} , and the evolution of ρ_{ee} is described by $\dot{\rho}_{ee} = -\dot{\rho}_{++} - \dot{\rho}_{--}$.

According to Eq. (13) the first line of Eq. (32) agrees with the differential equation of a two-level atom with levels $|+\rangle$ and $|e\rangle$. Because of Eqs. (24) and (30) and $\omega_{e-} > 0$ the second line in Eq. (32) is a new acceleration-induced effect typical for the multilevel situation. It vanishes for $a=0$. The population ρ_{++} of the $|+\rangle$ state is coupled to the coherences ρ_{+-} between the two ground states. The transition matrix elements \vec{d}_{+e} and \vec{d}_{-e} between each of the lower states and the excited state appear so that we have something similar to a Raman transition between the two not directly coupled ground states. On the other hand, the contributions of the two-point functions contained in $K(\omega_{e-})$ are those of the two-level system with states $|e\rangle$ and $|-\rangle$. After renormalization of $\text{Im}K(\omega)$ as done above, $K(\omega_{e-})$ is given by Eqs. (24) and (30). This has the direct consequence that not only the first term in Eq. (32) but also the last two terms are of the order of the Unruh effect for two-level atoms. Accordingly, although new effects appear, with regard to an experimental verification of the influence of acceleration, the situation is the same as in the two-level case. In this sense the Unruh effect is not enhanced.

This reasoning holds also in the general multilevel case. Considering Eq. (11) one can see that the argument of any of the functions $K_{ij,kl}(\omega)$ is always the frequency difference ω_{kl} of two states $|k\rangle$ and $|l\rangle$, which are coupled by a dipole moment d_{kl}^a . Hence, the transition rate is always the same as the one between the two states $|k\rangle$ and $|l\rangle$. We have as an important result that the magnitude of the acceleration-induced transition rates for the multilevel atom are essentially the same as for a two-level atom consisting of the states $|k\rangle$ and $|l\rangle$.

Let us now turn to another observation. Under special circumstances this multilevel effect has the interesting consequence that a linear combination of the two ground states may remain stable even in the presence of acceleration. For the Λ system at hand this happens if the dipole moments fulfill the condition $|\vec{d}_{+e}|^2 = |\vec{d}_{-e}|^2 = |\vec{d}_{+e} \cdot \vec{d}_{-e}|$. The *non-coupled state* then can be written as

$$|\psi_{\text{nc}}\rangle = \frac{1}{\sqrt{2}}\{|+\rangle - e^{-i\varphi}|-\rangle\}, \quad (34)$$

where the phase φ is given by $\arg(\vec{d}_{+e} \cdot \vec{d}_{-e})$. It is not difficult to show that the corresponding density matrix $\rho = |\psi_{\text{nc}}\rangle\langle\psi_{\text{nc}}|$ is time independent so that there is no acceleration-induced spontaneous excitation that makes this state unstable. The Unruh effect appears only for the *coupled state*

$$|\psi_c\rangle = \frac{1}{\sqrt{2}}\{|+\rangle + e^{-i\varphi}|-\rangle\}, \quad (35)$$

for which spontaneous excitation is present.

The existence of coupled and noncoupled ground states is well known in the field of atom optics and has led to an

intense research in laser cooling of atoms (see, e.g., Ref. [24]) and dark optical lattices (see, e.g., Ref. [25]). However, while in this field the coupling to laser beams with given intensity and frequency is concerned, we are here dealing with the coupling to the vacuum modes in the presence of acceleration, which acts similar to a thermal bath. We therefore conclude that although the Unruh effect in general affects the stability of atomic levels there is the possibility that the transition amplitudes of two different ground states interfere destructively so that a combination of them may remain stable.

It should be mentioned that stable states in the presence of thermal radiation are also principally possible. This follows because the fact that $|\psi_{\text{nc}}\rangle$ is a stationary solution of Eqs. (32), (33) does not depend on the precise form of $K(\omega)$ so that it might also stand for the thermal Green function. Though to our knowledge there is no real *free* atomic system fulfilling the condition on the dipole moments given above it should be possible to use external fields that induce dipole moments fulfilling the desired relation [26].

Three-level atom in V configuration: Let us finally follow another strategy that presents itself in connection with multilevel atoms. Are there certain stable atomic configurations where the stability depends crucially on the fact that the atom moves inertially? We study a possible candidate for such a situation. In the V configuration of a three-level atom we have one ground state $|g\rangle$ and two excited states $|\pm\rangle$ for which we will assume $E_+ = E_-$. The excited states are not directly coupled so that only $d_{g+}^a = d_{+g}^{a*}$ and $d_{g-}^a = d_{-g}^{a*}$ do not vanish. Because the two emission amplitudes from each excited level to the ground state can interfere destructively, the total spontaneous emission rate was observed to be canceled if the two excited states are coupled to a fourth level by a coherent driving field [27]. For the sake of simplicity we will consider here the unrealistic case that $d_{-g}^a = -d_{+g}^a$ holds and for which theoretically spontaneous emission cancellation occurs. Abbreviating $K(\pm\omega_{+g})$ by K_{\pm} the master equation (11) for the excited states can be found to be

$$\frac{d\rho_{++}}{dt} = \frac{|\vec{d}_{g+}|^2}{\hbar^2} \{2\rho_{gg}\text{Re}K_+ - 2\rho_{++}\text{Re}K_- + \rho_{+-}K_- + \rho_{+-}K_-^*\}, \quad (36)$$

$$\frac{d\rho_{--}}{dt} = \frac{|\vec{d}_{g+}|^2}{\hbar^2} \{2\rho_{gg}\text{Re}K_+ - 2\rho_{--}\text{Re}K_- + \rho_{+-}K_- + \rho_{+-}K_-^*\}, \quad (37)$$

$$\frac{d\rho_{+-}}{dt} = \frac{-|\vec{d}_{g+}|^2}{\hbar^2} \{2\rho_{+-}\text{Re}K_- + 2\rho_{gg}\text{Re}K_+ - \rho_{--}K_- - \rho_{++}K_-^*\}. \quad (38)$$

All we need to know about the other equations is that $d\rho_{gg}/dt = -d\rho_{++}/dt - d\rho_{--}/dt$ and that the master equation for the ground state coherences do not depend on $\rho_{\pm\pm}$ and $\rho_{\pm\mp}$. It is not difficult to see that these equations have the stationary solution $\rho_{++} = \rho_{--} = \rho_{+-} = \rho_{-+} = 1/2$ corresponding to the pure excited state $|\psi\rangle = \{|+\rangle + |-\rangle\}/\sqrt{2}$.

Since this solution is independent on the precise form of $K(\omega)$ we see that the atomic center-of-mass motion has no influence on the stability of this stable excited state although the individual decay rates may change.

VI. DISCUSSION

We have calculated the spontaneous transition rates of multilevel atoms, coupled to the electromagnetic field, which move with constant acceleration through the Minkowski vacuum. Corrections to the inertial rates become important only for huge accelerations. For multilevel systems new acceleration-induced transitions occur, but they do not alter the magnitude of the usual Unruh effect. For a three-level atom in the Λ configuration these transitions resemble Raman transitions, but in this case they are spontaneous because no photons are present in the Minkowski vacuum.

It is a common belief that the spontaneous processes induced by the accelerated motion lead to a general instability

of all the states of the multilevel atom. We have shown that in contrast to this, specific linear superpositions of the two ground states of a Λ system represent a noncoupled pure state that is stable, i.e., shows no Unruh effect.

In addition, a three-level atom in the V configuration has a state that is a noncoupled excited state if the atom is moving inertially. This stability *is not changed* in the case of accelerated motion, so that there is again no influence.

The influence of acceleration on the Lamb shift, although much too small to be measured, is always there. We have obtained the respective expression for the electromagnetic coupling after the usual renormalization.

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